On uniqueness of fibrous materials

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Abstract

This paper summarizes some remarkable issues in fibrous materials, i.e., any material systems formed by fiber-like constituents such as felt, cloth, paper, muscle and wood. Such interest can go back as far as Galileo who was fascinated by the fact that short fibres can form a long and strong rope via friction between fibers induced by twisting, although a relatively rigorous account for the mechanism has not been available until recently. It is almost exclusively the feature of fibrous materials where pliability, compressibility, high toughness and durability coexist, making the material irreplaceable and indispensable in many engineering applications. The questions regarding fiber packing, flexibility, friction, and fabric formability have inspirited studies not only from fiber scientists, but from science communities at large. Furthermore, the fiber capillary phenomena and the distinct thermal behaviours of the material have provoked many scientific curiosities and endeavours, yielding countless practical applications. Nonetheless, the fibrous materials, arguably the first type of engineering materials, remain perhaps the least understood, not to mention the fact that most biomaterials, plants or body muscles, are formed by fibrous constituents.
Keywords: fibrous materials, packing problem, friction, flexibility and formability, transport properties.

1   Fibrous materials

In a lesser but similar degree, we have been taking fibrous materials, just like air or water, for granted; they are so critically indispensable for us that we rarely pause to think about them from a materials science point of view. We expect our cloth or other textiles (one major portion of the fibrous materials) to be soft, pliable, with desirable durability, comfort yet not too heavy; but seldom to wonder why and how fibrous materials are able to offer such wonderful functions. The seeming ignorance from the scientific community at large has lead to current bewildering situation that, the fibrous materials, arguably the first type of designed engineering materials, remain perhaps the least understood.

The present article is just an attempt to highlight some unique attributes of fibrous materials and to explain, as well as the author could, the science behind them. Such an account should be useful even now giving the burgeoning applications of fibrous materials in new areas such as fibre reinforced composites in engineering and fibre-based products in medical and other biological fields; after all most biomaterials, plants or body muscles, are formed by fibrous constituents.

From materials science, fibrous structures can be viewed as a mixture of fiber and air, and are not classical continuum, inherently discrete due to the existence of the macro-pores. The deformations at the micro and macro levels often are of different nature; for instance, when you sitting on a thick wool felt, you are compressing the material; but closer examination will reveal that the individual fiber is actually experiencing typically bending deformation. Therefore, the connections between the formulations from the microstructural analysis and the macroscopic performance have to be established as the premises for the discrete media study.

Owing to the intrinsic random nature of the physical and geometrical features of distinct properties, fibrous materials mostly are not isotropic. They respond differently when loaded at different directions. Also, various loads have to be transferred through the fiber-fiber contact points. Therefore the format in which the fibers are arranged in a structure becomes a critical issue; it will determine virtually all the properties of the system.

2. Fiber packing problem

It is very logical that research on fibrous materials should start from the fiber packing problem, since it deals with the issue of how numerous individual fibers arranged or assembled to form the materials of different types. Fiber packing formats in terms of the system geometrical features are the critical attributes for any practical textile structure, and they will determine each of the individual physical properties of the materials including the mechanical and fluid transport behaviors. The problem of fiber packing was initially studied by Van Wyk in his analysis of the compressibility of wool by looking into the geometrical characteristics of a fiber mass formed by fiber packing [1]. Komori and colleagues have examined the details of such geometrical features in a fiber

assembly, including the mean fiber contact density and the mean fiber length between two fiber contact [2], and the fiber and pores distribution in the fiber assembly [3]. Pan has taken into consideration of the steric hindrance effect, i.e., the interference of existing fiber contacts on the successive new contact to be made [4]. So for a fiber mass of volume V, the mean fiber contacts n_l per unit length are

$$n_l = \frac{8sIV_f}{l_f(\pi + 4V_f\Psi)} \tag{2.1}$$

where l_f, V_f and $s = l_f/D_f$ are the fiber length, the fiber volume fraction and the fiber aspect ratio, respectively; I and Ψ are the two factors accounting for both the fiber orientation and the steric hindrance effect, which can be readily calculated once the fiber orientation density function is given. The reciprocal of n_l provides the mean length between two fiber contact points.

The total number of contacts in this fiber mass is

$$n = \frac{16IVV_f^2}{\pi s l_f^3(\pi + 4V_f\Psi)} \tag{2.2}$$

Note the important roles played by the fiber length l_f and the fiber volume fraction V_f [4].

For a given volume of the fiber mass, there are two competing factors affecting fiber contact. On one hand, an existing contact reduces the effective contact length of a given fiber and hence diminishes the chance for new contacts (the steric hindrance effect). On the other hand, the existing fiber contact point will also abate the free volume of the fiber mass, and consequently increase the chance for successive fibers to make new contacts. Some of the research results in this area have been applied to study the compressional [5] and shear [6] behaviors of general fiber assemblies, leading to considerable progress in those areas. Furthermore, fiber packing problem has also been studied in fiber reinforced composite materials [7, 8, 9].

The research on this problem is still very primitive. However, a thorough study of a structure formed by individual fibers is extremely challenging. It is worth mentioning that the problem of the micro-geometry in a fibrous material can be categorized into a branch of complex problems in mathematics called Packing problems. Take for example the sphere packing problem, also known as the Kepler problem, which has been an active area of research for mathematicians ever since it was first posed some three hundred years ago, and remains unsolved until even today [10]. Yet, it seems to me the sphere packing should be the simplest packing case, for one only needs to consider one parameter, i.e., the diameter of the spheres, and ignore the deformation due to packing. Therefore it doesn't seem to be the case that the fiber packing problem, with fiber length and diameter highly prone to deformation, can be solved completely anytime soon.

3. Flexible at will

One of the main features making fibrous structures the top choice in many applications has to do with its bending flexibility while still maintaining high tensile stiffness and strength; hence the seemingly eternal popularity of ropes. In comparison, a rod of equal diameter and material will be too rigid to even bend into a bow, let along to tie a knot. The difference lies in the deformation mechanism. In a rope under bending or shearing load, individual fibers in the rope will slide or move over each other, given rooms for individual fibers to bend into desired shapes. In other word, the fiber is bent in respect to its own axis of radius r; whereas in the rod, the bending rigidity is much greater because of the greater diameter or bending moment of inertia. Ideally, a rope of N fibers co-axially arranged only has, ignoring the inter-fiber friction, a bending moment of inertia

$$I_1 = N \frac{\pi r^4}{4} \tag{3.1}$$

Where as for a rod of equal mass, the radius R would be

$$R = \sqrt{N}\, r \tag{3.2}$$

So the corresponding bending moment of inertia is

$$I_2 = \frac{\pi R^4}{4} = N^2 \frac{\pi r^4}{4} = NI_1 \tag{3.3}$$

That is, the ratio of the two bending moments of inertia would be N. In other words, the rope will be N times more flexible under bending than the rod. Nevertheless, as we increase the twist on the rope, lateral pressure rises to reduce the relative mobility among the fibers as discussed in detail next section, and the rope would behave stiffer.

4. Twist and friction

Friction is the only mechanism by which fibrous materials are formed. In ropes or yarns, the friction is brought into play via tension on fibers of helical conformation due to twist. Whereas in a piece of fabrics, the friction takes place at the interlacing points of yarns crimped after weaving process to accommodate the perpendicular counterparts. This crimp serves the same critical purpose as helices in a yarn to provide pressure upon stretching so as to enhance the fabric.

Galileo [11] was fascinated by the fact that short fibres can form a long and strong rope via friction between fibers induced by twisting, although a relatively rigorous account for the mechanism has not been available until recently [12, 13]. Staple (short) fibers are assembled into a continuous strand (yarn) by virtue of twist, which leads to a spacious helical conformation of individual fibers in the yarn. Upon stretching, the tension on the helical fibers will generate lateral pressure to bind the fibers together to sustain the stretching as described by Hearle [14]. If the external stretching is non-existent, the yarn is just a loose structure of collected fibers held together by the weak adhesion and possible

fiber entangles; the yarn has virtually no strength. So it is truly fascinating that the very stretching which attempts to break the yarn is in fact reinforcing the yarn simultaneously. The twist (the fiber helicity) level obviously determines the ultimate outcome.

Upon stretching, the tension in the fiber is built up from zero at the fiber ends to the maximum somewhere along the fiber length, ideally at the center. The tension distribution along the fiber length is linear at the portion of fiber length where slippage occurs. But at the portion tightly gripped via inter-fiber friction, a hyperbolic tension distribution has been derived by Pan [12]. The distribution of the friction-generated shear stress within a yarn was also developed. As we increase the twist level to a critical point, a self-locking mechanism takes place where fibers are no longer slide over each other in a tensioned yarn. Instead, they bind each other to form a thread will considerable strength. Considering a fiber with both slipping ends of length $\frac{1}{2}\lambda l_f$, Pan [13] has proposed a relationship between this slipping proportionality λ and other related factors as

$$\lambda = \frac{Tanh(\mu n \bar{s})}{\mu n \bar{s}} \qquad (4.1)$$

where n is a factor determined by and increases with the twist level alone for a given yarn system, termed hence the dimensionless twist, the revised fiber aspect ratio $\bar{s} = s(1-\lambda)$, the fiber aspect ratio $s = \frac{l_f}{D_f}$, l_f the fiber length and D_f the diameter, μ the fiber-fiber frictional coefficient.

When λ equals to 1, the fiber is completely slipping. As soon as the yarn twist level reaches a critical point, the λ value will drop, revealing that the central portion of the fiber is gripped tightly, which in tern leads further reduction of λ until the whole fiber is held over its entire length, and a self-locking mechanism forms. This whole process in an ideal or a variation-free case would take place abruptly as predicted by eqn. (4.1) and plotted in Figure 4.1.

However, several complex problems have yet to be solved. First, in all the existing analysis, fiber to fiber contact in a yarn is assumed to be line contact. Yet in more realistic cases, fibers are in discrete point contact. This will completely alter the distributions of both the tensile and shear stresses in individual fibers. Also, several competing factors are involved in prediction of the optimal twist level at which a staple yarn acquires the maximum strength, including the twist level, the fiber volume fraction of the yarn, the statistical variations and the complex yarn fracture behavior as discussed in [15].
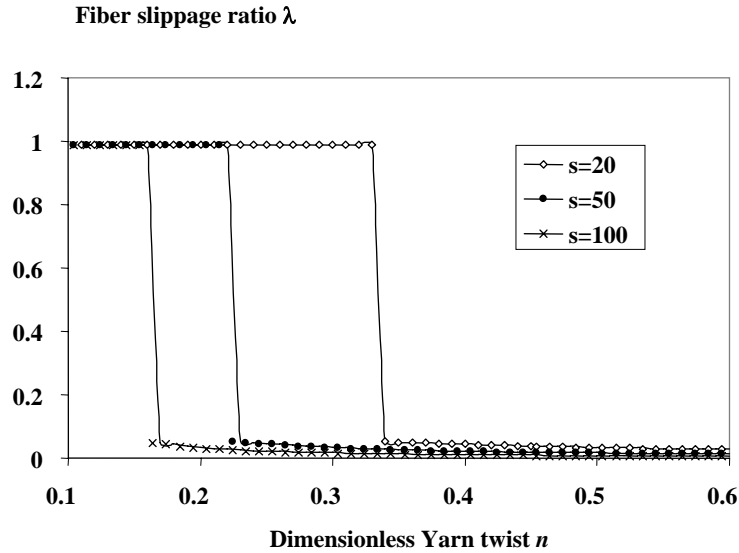
Fiber slippage ratio λ

Figure 4.1: Fiber slippage ratio λ vs. the dimensionless yarn twist *n*.

Knowing the critical role twist plays in generating yarn strength, people tend to think yarn twist to fabric strength is just as important as to yarn strength. Actually, it turned out that a woven fabric with twist-less yarns can be made, which possesses strength at par with ordinary wove fabrics of comparable types. So making fabrics using twisted yarn is mainly to facilitate the weaving process (e.g., preventing individual fibers from fraying away). Once yarns are in the fabric, it is the interlacing points where the fibers are held together via friction. In other words, even though twist is not a decisive factor in fabric strength, but friction remains the key in not only fabric, but in all other textiles. This makes textiles the most efficient, convenient and even smart materials.

5. Beauty of drape

No any other solid sheets or films can fit to a human body or other solid objects as elegantly as textile fabrics. Several factors contribute to this attribute. First, the relative movement of the structural components over each other during fabric deformation allows the multi-curvature bending, clearly unique only to fabrics and critical for its formability as studied by Hearle [16]. Another important factor is the unique response of fabrics to different types of stresses.

For an isotropic material, there is a simple relationship between the three parameters required to define the mechanical behaviour of the material,

$$G = \frac{E}{2(1+v)} \tag{5.1}$$

where E and G are the tensile and shear moduli respectively and ν is the Poisson's ratio of the material. For normal solid materials, $0 < \nu < 0.5$ so that

$$2 < \frac{E}{G} < 3 \qquad (5.2)$$

In other words, the tensile and shear moduli are in the same order of magnitude. Whatever the nature of the stress, the resistance of the material to deformation is no big difference. However or fortunately, this is not the case for fabrics, it is reported [17] that for fabrics

$$\frac{E}{G} \to 200 \qquad (5.3)$$

depending on the type of the fabric. That is, the fabric will shear easily even due to its own weight. Once a fabric is laid onto an object, it will deform in bending and shear until it covers the object to the degree allowable by this E/G ratio. Or in other words, the fabric formability is mainly due to the relative movement of individual yarns in the fabric subjected to a shear load. Under such load, the yarns will reorient through sliding and slipping towards the loading direction. This freedom of relative movement of yarns when the fabric is under bias extension is the key to offer a very low bending and shear resistance, leading to an unusually high E/G ratio or an excellent formability.

6. Unique wetting behavior of fibrous materials

According to Brochard [18], the so-called Harkinson spreading parameter S can be defined as

$$S = \gamma_{SO} - \gamma_{SL} - \gamma \qquad (6.1)$$

where γ_{SO}, γ_{SL} and γ represent the surface tensions of a solid fiber, a solid/liquid interface, and a liquid (or liquid/air). Then the fiber of radius r will be completely wetted by the liquid of thickness e when

$$S > \frac{e\gamma}{r} \quad . \qquad (6.2)$$

Whereas for complete wetting of a flat solid, it only requires

$$S > 0 \qquad (6.3)$$

From eqns. (6.2) and (6.3) we see that it is obvious that liquids will wet a solid plane more promptly than wet a fiber, or compared to the wetting of planes, the wetting of individual fibers is a more energy- consuming process.

Next, let us examine the case of a fiber bundle formed by n parallel fibers of radius r as seen in Fig. (4.1). For a length L of the dry fiber bundle, its surface energy equals to

$$W_b = 2\pi r n L \gamma_{SO} \qquad (6.4)$$

Once covered by the liquid of radius R, the fiber bundle has the surface energy

$$W_m = 2\pi r n L \gamma_{SL} + 2\pi R L \gamma \qquad (6.5)$$

That is, the energy W_m is composed of both terms of solid/liquid interface and liquid/air interface. The complete wetting sets in when the wet state of the system is energetically more favourable compared with the dry one, i.e., when $W_b > W_m$. Or, from previous equations

$$\gamma_{SO} - \gamma_{SL} - \frac{R\gamma}{nr} > 0 \tag{6.6}$$

Inserting Harkinson spreading coefficient from eqns. (6.1) into (6.6) yields

$$S > \frac{R - nr}{nr}\gamma \tag{6.7}$$

So the critical value S_{Cb} for the complete wetting of the bundle system is

$$S_{Cb} = \frac{R - nr}{nr}\gamma \tag{6.8}$$

The liquid radius R could be smaller than the total sum of fibers radii nr. Figure 6.1 shows such an example when the cross-section of the 7-fiber bundle is covered by a liquid cylinder $R=3r$. The value of S_{Cb} is only $-4/7\ \gamma < 0$.
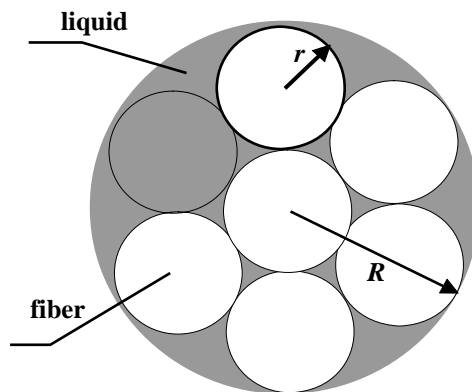

liquid

r

fiber

R

Figure 6.1: the cross-section of a *7*-fiber bundle covered by a liquid cylinder
$R=3r$.

The above results show that, on one hand, the liquid will wet a solid plane but not a single fiber of the same material. On the other hand, the liquid will wet a fiber bundle even before it does the solid plane. The above simple analysis explains the excellent wetting properties of a fiber mass, the familiar capillary phenomenon, in terms of energy changes. Furthermore, the consequence of the collective behaviour of fibers in the bundle allows the energy W_m increasing more rapidly with the fiber number n in the bundle than the dry bundle energy W_b.

7. Thermal resistivity versus the fiber volume fraction

By treating a fibrous material as a mixture of fibers and the still air trapped inside the pores formed by the fibers, the system behaviour of the material thus becomes a resultant of those of the two constituents. Since air plays a much greater role in thermal case than it does in mechanics, some kind of rule of mixtures seems more appropriate here to deal with the thermal behaviour of the fibrous materials. The simplest form of the rule of mixtures would be

$$\lambda = V_f \lambda_f + V_{sa}\lambda_{sa} = V_f \lambda_f + (1 - V_f)\lambda_{sa} \qquad (7.1)$$

Where λ is termed as the overall intrinsic thermal resistivity of the system; and λ_i and V_i $(i= f,\ sa)$ are the thermal resistivity and the volume fraction for the constituent i. For an ideal case, the dashed straight line in Fig. 7.1 plotted based on Eqn. (7.1) would offer a simple answer. However, it is of common sense that many more factors or mechanisms are involved in heat transport process such as conduction, convection via flowing air, radiation and phase change. Consequently, the system λ value is a function of all these factors as well. To further complicate the matter, the relative contribution of each factor is more likely a function of the fiber volume fraction V_f. For instance, as we increase the amount of the fiber into the system of given volume, we block convection, facilitate conduction and alter the radiation, leading to changing of the internal thermal energy of the system, which in turn causes the phase change of the moisture in the system. If considering the fact that measurements are normally done using the hot plate method where a temperature gradient ΔT is applied on both sides of the material, several complex scenarios are expected that we even cannot plot a complete curve. First, the lines in Fig. 7.1 becomes non-linear, or even non-monotonic; the maxima or minima no longer occur at the two ends where V_f =0 or 1. It is also interesting to contemplate the possibility of chaotic or even singular points somewhere over the full ranges.
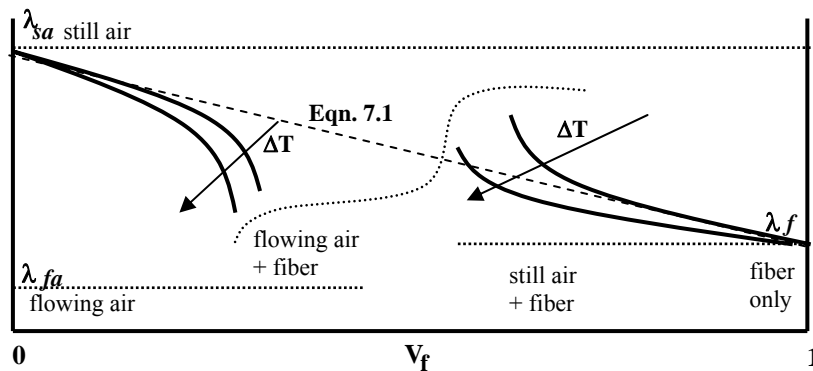

Figure 7.1: Thermal resistance of fibrous materials vs. fiber volume fraction (measured via hot plate method).

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