

# Mechanical properties of a woven fabric

Huiyu Sun\* Ning Pan

Department of Biological and Agricultural Engineering, Division of Textiles and Clothing, University of California, Davis, CA 95616, USA

---

## Abstract

This paper, introducing some recent research progress, consists of two parts: the shear deformation analysis and Poisson's ratios for woven fabrics. The analytical methods of the shear moduli and Poisson's ratios for woven fabrics will enable more rigorous studies on such important issues of fabric bending and draping behaviors.

*Keywords:* Plain weave; Shear deformation; Undulation geometry; Poisson's ratios

---

## 1. Introduction

In undergoing an investigation of fabric shearing, it was initially suggested by Mack and Taylor [1] that a fabric behaves as a pin-jointed mesh, whereby the tows rotate or trellis about the fabric cross-over. By further assuming that the yarns are inextensible and incompressible, Grosberg and Park [2] proceeded to analyze the modes of deformation involved in shearing of plain woven fabrics in terms of the mechanical properties of yarns and the geometrical parameters of the fabric. Using fabric geometric and material parameters, Nguyen et al. [3] predicted the initial slip region of the fabric as well as the more dominant elastic deformation range.

The Poisson's ratio is one of the fundamental properties of any engineering materials, and represents important mechanical characteristics for a woven fabric in many applications including in engineering systems that incorporate textile fabrics as structural elements. Hearle et al. [4] derived the Poisson's ratio of a woven fabric assuming that the yarn extension and compression were negligible. By modeling the individual yarn as *extensible* elastica, Warren [5] in 1990 determined the in-plane linear elastic constants of woven fabrics. Results of this theoretical analysis compare favorably with the measured in-plane Young's moduli of woven fabrics.

## 2. Theoretical analysis

### 2.1. Analysis for shear deformation

The geometry of the woven fabric under consideration here is shown in Fig.1. The initial unloaded yarn geometry is assumed to be a sequence of alternating circular arcs of constant radius  $R$ . With reference to Fig.1 (a), the usual geometrical weave parameters of pick spacing  $P$ , yarn length  $l$  and crimp height  $h$  can be represented in terms of the radius  $R$  and crimp angle  $\phi_o$ . The angle  $\alpha$  corresponds to the non-contact segment of curved yarns, the angle  $\beta$  corresponds to the contact-segment of yarns with a length of  $\frac{d}{2}$ , and  $d$  is the contact length of yarns.

#### 2.1.1. Deformation due to rigid intersections

If a shear force is applied which is less than the frictional restraint at the intersecting points, the fabric should behave like an elastic grid structure whose joints are welded at the contact points. Because the joint behaves as if welded for these extremely small deformations (order of 0.05 degree of shearing), each beam deforms as a cantilever. With reference to Fig.1(b), the bending and torsional moments at any cross-section  $D$  within the non-contact segment of curved yarns with an angle  $\varsigma$  due to the external force  $f$  and the imaginary external couple  $M_a$  applied at the end of the curved cantilever are:

$$M_x = -fR\sin(\alpha - \varphi) - M_a\cos(\alpha - \varphi) \quad (1a)$$

$$M_z = -fR[1 - \cos(\alpha - \varphi)] + M_a\sin(\alpha - \varphi) \quad (1b)$$

---

\* Corresponding author. Tel.: + 530 752 8984; Fax: + 530 752 7584; E-mail: hsun@uedavis.edu.

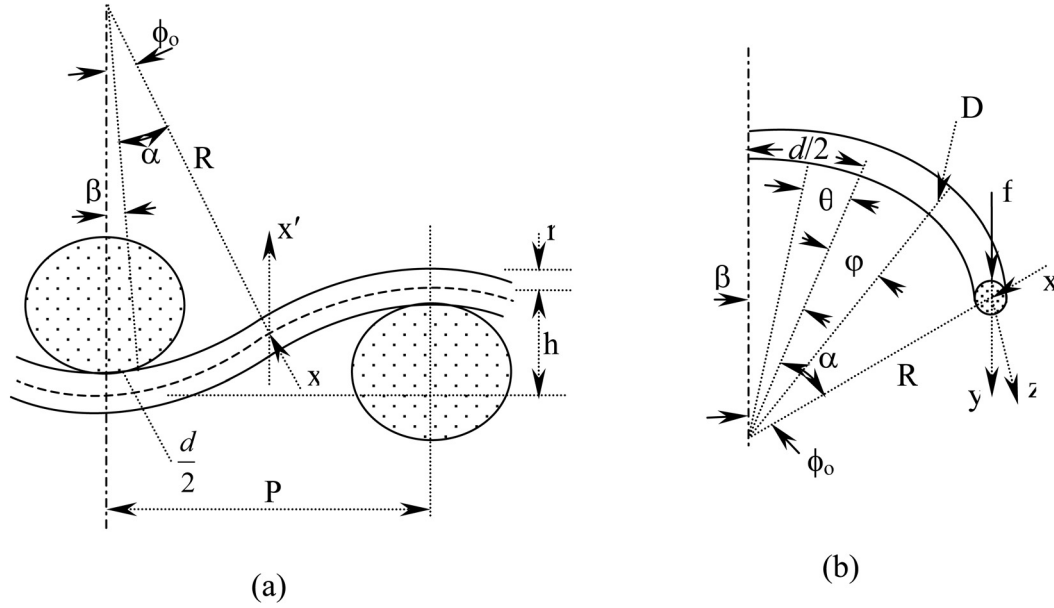


Fig.1. (a) Cross-section of weave in either warp or weft direction. (b) Schematic of an undulant yarn loaded by a force  $f$ .

The strain energy for the curved non-contact segment of yarns can be determined as:

$$U = \int_0^\alpha \left( \frac{M_x^2}{2EI_x} + \frac{M_z^2}{2C} \right) R d\varphi \quad (2)$$

where  $E$  is the elastic modulus of the yarn,  $I_x$  is the inertia moment of the yarn cross-section about the axis  $x$ ,  $C$  is the torsional rigidity about the axis  $z$ .

According to the theorem of Castigliano, the rotational angle about the axis  $x$  at the end of the curved cantilever is given by:

$$\delta = \frac{\partial U}{\partial M_a} \Big|_{M_a=0} \quad (3)$$

Substituting Formula (1) into (3) leads to:

$$\begin{aligned} \delta &= \int_0^\alpha \left[ \frac{M_x}{EI_x} \frac{\partial M_x}{\partial M_a} + \frac{M_z}{C} \frac{\partial M_z}{\partial M_a} \right] R d\varphi \Big|_{M_a=0} = \\ &= \int_0^\alpha \left[ \frac{fR \sin(\alpha - \varphi)}{EI_x} \cos(\alpha - \varphi) + \right. \\ &\quad \left. \frac{fR[1 - \cos(\alpha - \varphi)]}{C} \sin(\alpha - \varphi) \right] R d\varphi = \\ &= fR^2 \left[ \frac{\sin^2 \alpha}{2EI_x} + \frac{(1 - \cos \alpha)^2}{2C} \right] \end{aligned} \quad (4)$$

The rotational angle about the axis  $x'$  (cf. Fig. 1) is obtained.

$$\delta' = fR^2 \cos \phi_0 \left[ \frac{\sin^2 \alpha}{2EI_x} + \frac{(1 - \cos \alpha)^2}{2C} \right] \quad (5)$$

### 2.1.2. Yarn slippage at the intersecting region

When the portion from  $0$  to  $\theta$  can slip (while the portion from  $\theta$  to  $\beta$  remains as if welded), the frictional moment is obtained as follows:

$$\begin{aligned} M &= \mu \int_0^\theta \frac{V\varphi}{\beta^2 R} \left[ 2R \sin\left(\frac{\theta - \varphi}{2}\right) \right] R d\varphi = \\ &= \frac{4\mu V R}{\beta^2} \left( \theta - 2 \sin \frac{\theta}{2} \right) \end{aligned} \quad (6)$$

where  $V$  is the normal contact force. For the problem tackled in this paper, in case the undulant radius of warp yarns is not equal to that of weft yarns, the frictional moment is taken as an average of the two possible values:

$$M = \frac{2\mu V(R_1 + R_2)}{\beta^2} \left( \theta - 2 \sin \frac{\theta}{2} \right) \quad (7)$$

where the subscripts 1 and 2 denote the warp and weft directions, respectively.

On the other hand, the external moment due to the shear force is also taken as an average of the two possible values, that is:

$$M = \frac{1}{2}(M_1 + M_2) = \frac{1}{2} \left[ f_2 \cdot 2R_1 \sin\left(\frac{\alpha_1 + \theta}{2}\right) + \right.$$

$$f_1 \cdot 2R_2 \sin\left(\frac{\alpha_2 + \theta}{2}\right) = \frac{F}{L} \left[ P_2 R_1 \sin\left(\frac{\alpha_1 + \theta}{2}\right) + P_1 R_2 \sin\left(\frac{\alpha_2 + \theta}{2}\right) \right] \quad (8)$$

Considering slippage equilibrium, the external moment should be equal to the frictional moment. From Formulas (7) and (8), the shear force is obtained.

$$F = \frac{\frac{2\mu V(R_1+R_2)}{\beta^2} (\theta - 2 \sin \frac{\theta}{2})}{\frac{1}{L} [P_2 R_1 \sin(\frac{\alpha_1+\theta}{2}) + P_1 R_2 \sin(\frac{\alpha_2+\theta}{2})]} \quad (9)$$

From Equ. (5), substituting  $(\alpha + \theta)$  for  $\alpha$ , the shear angle  $\gamma$  is got.

$$\begin{aligned} \gamma &= \delta'_1 + \delta'_2 \\ &= \frac{F}{L} \left\{ P_2 R_1^2 \cos \phi_{o1} \left[ \frac{\sin^2(\alpha_1 + \theta)}{2E_1 I_{x1}} + \frac{[1 - \cos(\alpha_1 + \theta)]^2}{2C_1} \right] + P_1 R_2^2 \cos \phi_{o2} \left[ \frac{\sin^2(\alpha_2 + \theta)}{2E_2 I_{x2}} + \frac{[1 - \cos(\alpha_2 + \theta)]^2}{2C_2} \right] \right\} \end{aligned} \quad (10)$$

Because yarn slippage takes place gradually from the outer boundary of the contact to the inner boundary,  $\theta$  varies from 0 to  $\beta$  in above formulas.

### 2.2. Analysis for Poisson's ratios

A woven fabric is composed of two sets of orthogonal interlaced yarns: warp and weft. Each yarn is modeled as extensible elastica, as shown in Fig. 2, where  $s_o$  is the original arc length along the undeformed curve of total length  $L_o$ ,  $\phi$  is the slope of the undeformed elastica,  $s$  is the arc length along the deformed curve having total length  $L$ ,  $\psi$  is the slope of deformed elastica. With the

force  $T_o$  applied at the end  $s = L$  and the reactions  $F$  (horizontal) and  $V$  (vertical) applied at the symmetry end  $s = 0$ , the differential equation describing the non-linear deformation of the extensible elastica is [5]:

$$EI \frac{d}{ds} \left[ \left( 1 + \frac{T}{EA} \right) \frac{d\psi}{ds} - \frac{d\phi}{ds} \right] = - \frac{dT}{ds} \quad (11)$$

where  $E$  is the Young's modulus of the yarn,  $I$  is the moment of inertia for the yarn cross-section,  $T$  is the axial force acting through the centroid of the yarn cross-section of area  $A$ . The boundary conditions for the differential equation are:

$$\psi(0) = 0 \quad \text{at the symmetry end} \quad (12a)$$

$$\left[ \left( 1 + \frac{T}{EA} \right) \frac{d\psi}{ds} \right]_{s=L} = \frac{d\phi}{ds} \Big|_{s_o=L_o} \quad \text{at the anti-symmetry end} \quad (12b)$$

Integrating the differential Eq. (11) twice with the boundary conditions leads to the displacements at the endpoint ( $s = L$ ) of the elastica. In the same way, the displacements at the anti-symmetry endpoint of the other set of yarns orthogonal to the axis  $x$  are determined. By applying the equilibrium that requires the contact force  $V$  to be the same for both warp and weft yarns, and geometric compatibility that requires the displacements in the vertical direction to be the same, the strain-stress relations are given [5]:

$$\begin{aligned} \epsilon_{xx} &= \frac{R_x^3}{2E_x I_x (\Gamma C_x + C_y)} [(\Gamma(A_x C_x - B_x^2) + A_x C_y) \\ &\quad \left( \frac{R_y \sin \phi_{oy}}{R_x \sin \phi_{ox}} \right) \sigma_{xx} - B_x B_y \sigma_{yy}] \end{aligned} \quad (13a)$$

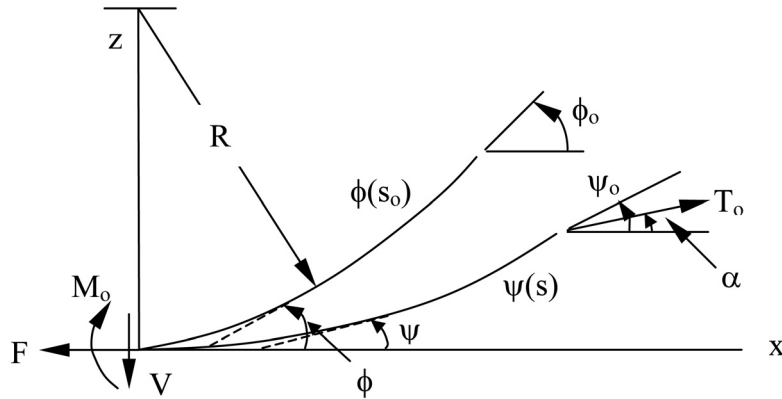


Fig.2. The extensible elastica.

Table 1  
Predicted and experimental shear moduli and angles of shear for the initial slip region

	Shear modulus (N/deg)			Slip region (deg)		
	Ref. [3]	This paper	Experiment	Ref. [3]	This paper	Experiment
PW (1K)	3.6	3.4	2.5	0 → 0.18	0 → 0.25	0 → 0.40
PW (3K)	1.9	1.8	1.4	0 → 0.25	0 → 0.35	0 → 0.35

Table 2  
Theoretical and experimental values of Poisson's ratios of plain woven fabrics

	$\nu_{xy}$		$\nu_{yx}$	
	Theoretical	Experimental [6]	Theoretical	Experimental [6]
Fabric 1	0.473	0.341	0.473	0.341
Fabric 2	0.433	0.202	0.433	0.202
Fabric 3	0.406	0.342	0.489	0.240
Fabric 4	0.380	0.438	0.533	0.567

$$\varepsilon_{yy} = \frac{R_y^3}{2E_y I_y (\Gamma C_x + C_y)} \left[ \left( \Gamma A_y C_x + (A_y C_y - B_y^2) \right) \left( \frac{R_x \sin \phi_{ox}}{R_y \sin \phi_{oy}} \right) \sigma_{yy} - \Gamma B_x B_y \sigma_{xx} \right] \quad (13b)$$

where  $A$ ,  $B$  and  $C$  are functions of geometric parameters of fabric structures, the subscripts  $x$  and  $y$  indicate warp and weft yarns, respectively, and

$$\Gamma = \frac{E_y I_y}{E_x I_x} \left( \frac{R_x}{R_y} \right)^3 \quad (14)$$

The Poisson's ratios for a woven fabric are thus determined as follows:

$$\nu_{xy} = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}} = \frac{R_x \sin \phi_{ox}}{R_y \sin \phi_{oy}} \frac{B_x B_y}{\Gamma (A_x C_x - B_x^2) + A_x C_y} \quad (15a)$$

$$\nu_{yx} = -\frac{\varepsilon_{xx}}{\varepsilon_{yy}} = \frac{R_y \sin \phi_{oy}}{R_x \sin \phi_{ox}} \frac{\Gamma B_x B_y}{\Gamma A_y C_x + (A_y C_y - B_y^2)} \quad (15b)$$

### 3. Results

Table 1 shows the average predicted shear modulus and slip region in comparison with the experimental values for this fabric deformation zone. The modeling results from this paper for both shear modulus and slip region are better than those in [3]. Yet, the model proposed in this paper needs to be further improved for more precise analysis for the shear deformation of woven fabrics.

For Poisson's ratios of woven fabrics, the

experimental measurements are excerpted from the [6]. The comparison of theoretical predictions with the measured results is listed in Table 2. In general, the analytical calculations are in a reasonable agreement with the measurements.

### 4. Conclusions

Considering the undulation of yarns in a fabric, a mechanical model for initial shear deformation is proposed in this paper. This model more closely reflects the reality of woven fabrics compared to the model by Grosberg and Park [2], and produces better agreement with experiments than the existing results.

The Poisson's ratio for a woven fabric is predicted by lifting the previous assumption that the yarn in the fabric is inextensible. Theoretical analysis compares favorably with the experimental results.

This study provides a guideline for the design of a woven fabric.

### References

- [1] Mack C, Taylor HM. The fitting of woven cloth to surfaces. *Journal of Textile Institute* 1956;47(2):T477–T488.
- [2] Grosberg P, Park BJ. The mechanical properties of woven fabrics, Part V: the initial modulus and the frictional restraint in shearing of plain weave fabrics. *Textile Research Journal* 1966;36:420–431.
- [3] Nguyen M, Herszberg I, Paton R. The shear properties of

- woven carbon fabric. *Composite Structures* 1999;**47**:767–779.
- [4] Hearle JWS, Grosberg P, Backer S. *Structural mechanics of fibers, yarns and fabrics*, Volume 1. New York: Wiley-Interscience 1969.
- [5] Warren WE. The elastic properties of woven polymeric fabric. *Polymer Engineering and Science* 1990;**30**(20):1309–1313.
- [6] Bao L, Takatera M, Shinohara A. Error evaluation on measuring the apparent Poisson's ratios of textile fabrics by uniaxial tensile test, *Sen'i Gakkaishi* 1997;**53**(1):20–26.