

A Nonlinear Dynamic Model for Two-Strand Yarn Spinning

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Abstract

A nonlinear dynamic model is established for two-strand spun or Sirospun yarns. The homotopy perturbation method proposed in our earlier work is used to calculate the approximate oscillating periods in the vertical and horizontal directions. The study reveals that the optimal convergence angle of the two strands in equilibrium is 90°, while when the convergence angle is near 127°, resonance occurs.

Two-strand or Sirospun yarns [13] are produced on a conventional ring frame by feeding two rovings, drafted simultaneously, into the apron zone at a predetermined separation. Emerging from the nip point of the front rollers, the two strands are twisted together to form a two-ply structure (see Figure 1), and the mechanical character of such a two-strand yarn can be dramatically improved over its parent yarns. Figure 1 illustrates a model of two-strand yarn spinning.

In our previous paper, we established a quasistatic model of two-strand yarn spinning to determine the convergence point of the strands [3] and a linear dynamic model for the problem [4]. In this paper we will establish a nonlinear model for the problem.

Nonlinear Dynamic Model

Assume that the convergence point (equilibrium position) leads to an instantaneous position (see Figure 2), and the distances x and y are measured from the equilibrium position. Thus, the motion equations in the x - and y -directions can be expressed as

$$M \frac{d^2x}{dt^2} + F_1 \cos \alpha - F_2 \cos \beta = 0 \quad (1)$$

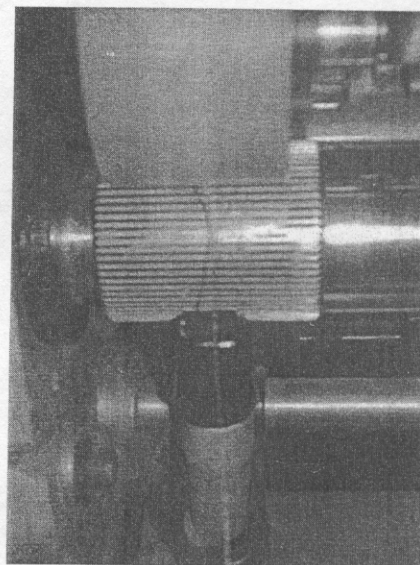


FIGURE 1. Two-strand yarn spinning.

$$M \frac{d^2y}{dt^2} + F_1 \sin \alpha + F_2 \sin \beta - F = 0 \quad (2)$$

Here M is the total mass of a fixed control volume $ABCD$ illustrated in Figure 3. The control volume is chosen in such a way that the mass center locates on the convergence point (O) of the two strands.

The mass M is determined from the following relation:

$$M = \rho_1 l_1 + \rho_2 l_2 + \rho h \quad (3)$$

where l_1 , l_2 and ρ_1 , ρ_2 are, respectively, the length and density per unit length of two parent strands above the

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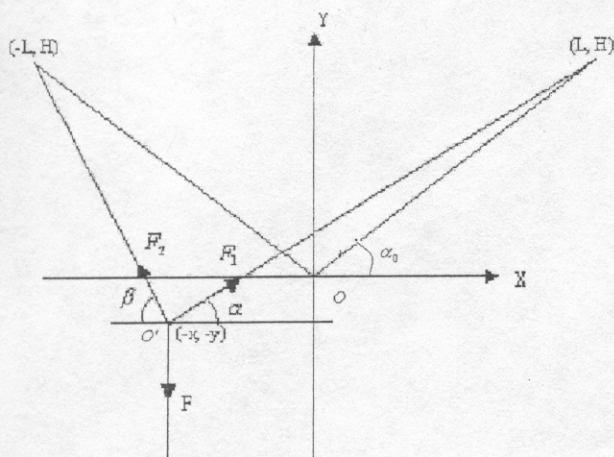


FIGURE 2. Dynamic illustration of a two-strand spun yarn.

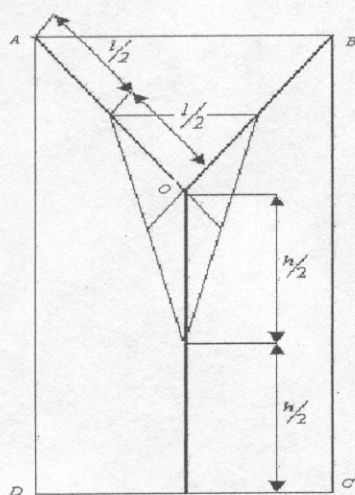


FIGURE 3. Control volume; the mass center locates at the convergence point.

convergence point, h is the distance of the two-strand yarn below the convergence point, which should be chosen such that in the mass center, the control volume $ABCD$ locates at the convergence point, and ρ is the density per unit length of the two-strand yarn.

Let the ends of the two strands above the convergence point be fixed a distance $2L$ apart, and let the equilibrium position be H . Thus, we have

$$\cos \alpha = \frac{L+x}{\sqrt{(L+x)^2 + (H+y)^2}} = L(L^2 + H^2)^{-1/2} + x(L^2 + H^2)^{-1/2} - L(xL + yH)(L^2 + H^2)^{-3/2} + \frac{1}{2}[-3L(L^2 + H^2)^{-3/2} + 3L^3(L^2 + H^2)^{-5/2}]x^2$$

$$+ \frac{1}{2}[-L(L^2 + H^2)^{-3/2} + 3LH^2(L^2 + H^2)^{-5/2}]y^2 + [-H(L^2 + H^2)^{-3/2} + 3L^2H(L^2 + H^2)^{-5/2}]xy, \quad (3)$$

$$\sin \alpha = H(L^2 + H^2)^{-1/2} - LH(L^2 + H^2)^{-3/2}x + y(L^2 + H^2)^{-1/2} - yH^2(L^2 + H^2)^{-3/2} + \frac{1}{2}[-H(L^2 + H^2)^{-3/2} + 3L^2H(L^2 + H^2)^{-5/2}]x^2 + \frac{1}{2}[-3H(L^2 + H^2)^{-3/2} + 3H^3(L^2 + H^2)^{-5/2}]y^2 + [-L(L^2 + H^2)^{-3/2} + 3LH^2(L^2 + H^2)^{-5/2}]xy, \quad (4)$$

$$\sin \beta = H(L^2 + H^2)^{-1/2} + LH(L^2 + H^2)^{-3/2}x + y(L^2 + H^2)^{-1/2} - yH^2(L^2 + H^2)^{-3/2} + \frac{1}{2}[-H(L^2 + H^2)^{-3/2} + 3L^2H(L^2 + H^2)^{-5/2}]x^2 + \frac{1}{2}[-3H(L^2 + H^2)^{-3/2} + 3H^3(L^2 + H^2)^{-5/2}]y^2 + [L(L^2 + H^2)^{-3/2} - 3LH^2(L^2 + H^2)^{-5/2}]xy, \quad (5)$$

$$\cos \beta = L(L^2 + H^2)^{-1/2} - (L^2 + H^2)^{-1/2}x + xL^2(L^2 + H^2)^{-3/2} - yLH(L^2 + H^2)^{-3/2} + \frac{1}{2}[-3L(L^2 + H^2)^{-3/2} + 3L^3(L^2 + H^2)^{-5/2}]x^2 + \frac{1}{2}[-L(L^2 + H^2)^{-3/2} + 3LH^2(L^2 + H^2)^{-5/2}]y^2 + [H(L^2 + H^2)^{-3/2} - 3L^2H(L^2 + H^2)^{-5/2}]xy. \quad (6)$$

In this paper, we consider a symmetrical, simple case, i.e., $\alpha_1 = \alpha_2$, $F_1 = F_2 = f$, when the system is in equilibrium. Integrating 3–6 into 1 and 2, we obtain

$$\frac{d^2x}{dt^2} + \omega_x^2 x + axy = 0, \quad (7)$$

$$\frac{d^2y}{dt^2} + \omega_y^2 y + bx^2 + cy^2 = 0, \quad (8)$$

where

$$\omega_x^2 = 2f[(L^2 + H^2)^{-1/2} - L^2(L^2 + H^2)^{-3/2}]M^{-1}, \quad (9)$$

$$\omega_y^2 = 2f[(L^2 + H^2)^{-1/2} - H^2(L^2 + H^2)^{-3/2}]M^{-1}, \quad (10)$$

$$a = 2f[-H(L^2 + H^2)^{-3/2} + 3L^2H(L^2 + H^2)^{-5/2}]M^{-1}, \quad (11)$$

$$b = f[-H(L^2 + H^2)^{-3/2} + 3L^2H(L^2 + H^2)^{-5/2}]M^{-1}, \quad (12)$$

$$c = f[-3H(L^2 + H^2)^{-3/2} + 3H^3(L^2 + H^2)^{-5/2}]M^{-1}. \quad (13)$$

There exist no small parameters in Equations 7 and 8, so the classical perturbation methods cannot be directly applied. Recently some new perturbation technologies have appeared that do not depend upon small parameters, for examples, the homotopy perturbation method [5, 6, 7], the modified Lindstedt-Poincare methods [8, 9, 10], the delta method [1, 2], and the energy balance method [11]. Much literature also exists on asymptotic methods [12], so our references to the literature will not be exhaustive. Rather, our purpose in this paper is to use the homotopy perturbation method to search for analytical solutions to Equations 7 and 8, which embody the essential relationships needed by engineers who have to design practical systems.

We construct a homotopy system in the following form:

$$\frac{d^2x}{dt^2} + \omega_x^2x + paxy = 0, \quad (14)$$

$$\frac{d^2y}{dt^2} + \omega_y^2y + p(bx^2 + cy^2) = 0. \quad (15)$$

It is obvious that when $p = 0$, the system of Equations 14 and 15 becomes the linearized one in reference 4, and when $p = 1$, it turns out to be the original system of Equations 7 and 8. The embedding parameter p monotonically increases from zero to a unit as the linear system ($p = 0$) is continuously deformed to the original system of Equations 7 and 8. So if we can construct an iteration formula for the system of Equations 14 and 15, a series of approximations appears as a solution by incrementing the imbedding parameter from zero to one; this continuously maps the initial solution of 14 and 15 into the solution of the original system of Equations 7 and 8.

According to the homotopy perturbation method [5, 6, 7], the solutions can be expressed in the following form:

$$x = x_0 + px_1 + p^2x_2 + \dots, \quad (16)$$

$$y = y_0 + py_1 + p^2y_2 + \dots. \quad (17)$$

Substituting 16 and 17 into 14 and 15, and collecting terms of the same power of p , we obtain the following differential equations for x_0 , x_1 and y_0 , y_1 :

$$\frac{d^2x_0}{dt^2} + \omega_x^2x_0 = 0, \quad (18)$$

$$\frac{d^2x_1}{dt^2} + \omega_x^2x_1 + x_0y_0 = 0, \quad (19)$$

$$\frac{d^2y_0}{dt^2} + \omega_y^2y_0 = 0, \quad (20)$$

$$\frac{d^2y_1}{dt^2} + \omega_y^2y_1 + ax_0^2 + by_0^2 = 0. \quad (21)$$

The solutions of 18 and 20 are, respectively,

$$x_0 = A \sin \omega_x t, \quad (22)$$

$$y_0 = B \sin \omega_y t, \quad (23)$$

where A and B are amplitudes in the x - and y -directions, respectively. Using the initial conditions for x_1 and y_1 — $x_1(0) = x_1'(0) = 0$, and $y_1(0) = y_1'(0) = 0$ —we can easily obtain the solutions for x_1 and y_1 . If the first-order approximates are enough, we place $p = 1$ in 16 and 17 to yield the following approximate solutions:

$$x = x_0 + x_1 = A \sin \omega_x t - \frac{AB}{2(\omega_x^2 - (\omega_x - \omega_y)^2)} [\cos(\omega_x - \omega_y)t - \cos \omega_x t] + \frac{AB}{2(\omega_x^2 - (\omega_x + \omega_y)^2)} \times [\cos(\omega_x + \omega_y)t - \cos \omega_x t], \quad (24)$$

$$y = y_0 + y_1 = B \sin \omega_y t - \frac{1}{2\omega_y^2} (aA^2 + bB^2) \times (1 - \cos \omega_y t) - \frac{aA^2}{2(4\omega_x^2 - \omega_y^2)} (\cos 2\omega_x t - \cos \omega_y t) - \frac{bB^2}{6\omega_y^2} (\cos 2\omega_y t - \cos \omega_y t). \quad (25)$$

If $2\omega_x = \omega_y$ (i.e., $L = 2H$ or $\alpha_0 = 26.56^\circ$), resonance occurs. This phenomenon should be completely avoided in textile applications.

Conclusions

We suggest a nonlinear dynamic model for two-strand spun yarns, which can be used directly by the textile industry. Our study reveals that the optimal convergence angle of the two strands in equilibrium is 90° , while when the convergence angle is nearly 127° , resonance occurs.

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