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A Linear Dynamic Model for Two-Strand Yarn Spinning

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ABSTRACT

A linear dynamic model is established for two-strand or Sirospun yarn processing. Approximate oscillating frequencies in the vertical and horizontal directions are obtained. By suitable choice of certain processing parameters, the mixture construction after the convergence point can be optimally matched.

Two-strand spun or Sirospun yarns are now widely used in the worsted industry. The strands are textured to improve the bulk of the resultant yarns, which possess more desirable properties. For example, the weaveability of fabrics formed by Sirospun yarns is significantly improved compared to their counterpart yarns.

Lately, many static models have been established to describe Sirospun yarn processing with the aid of experimental data to make the models closed [1, 2, 4, 5]. Yet our recent study [3] reveals that no experimental data are needed to definitely identify such system parameters as the strand convergence point or the convergence angles. The system should obey not only the force balance, which has been studied extensively in the open literature, but also all the conservation equations (*i.e.*, conservation of mass, momentum, and energy), so the quasistatic model thus suggested by our group [3] is self-closed.

In this paper, we develop a dynamic model for this problem. The characteristics of two-strand spun yarns depend mainly on how the two strands are combined and mixed, and the different oscillating frequencies of the convergence points in both the vertical and horizontal directions prove to be the dominant factors.

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Linear Dynamic Model

Figure 1 illustrates two-strand yarn spinning as an asymmetric case, and Figure 2 is the actual experimental set-up. We first assume the system is in a stable condition. With our quasistatic model [3], we can determine the convergence point with ease. Due to some perturbations, the convergence point (equilibrium position, O in Figure 3) moves to an instantaneous new position (O'). The distances x and y are measured from the equilibrium position. From Figure 3, it is easy to see that the projections in the x- and y-directions of the forces F in the two-strand yarn below the convergence point and F_1 and F_2 in the two strands above the convergence point are, respectively, 0, $-F$; $F_1 \cos \alpha$, $F_1 \sin \alpha$; and $-F_2 \cos \beta$, $F_2 \sin \beta$, where the angles α and β are defined in Figure 3.

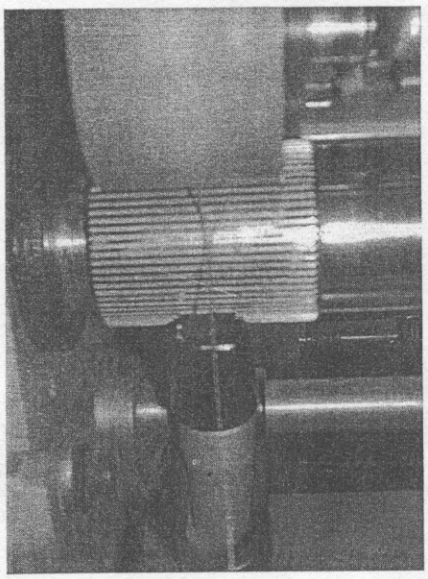

FIGURE 2. Experiment set-up.

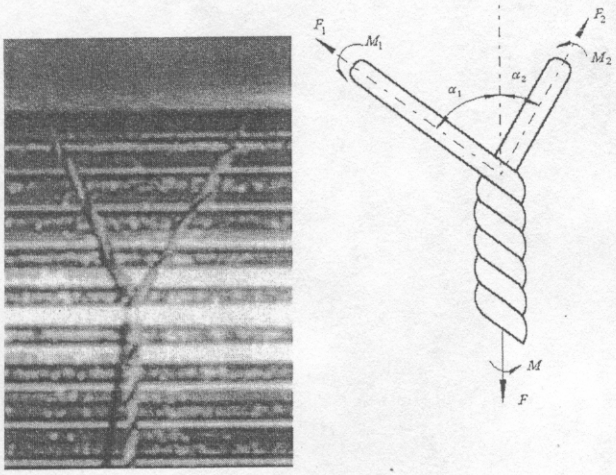

FIGURE 1. Asymmetric two-strand yarn spinning.

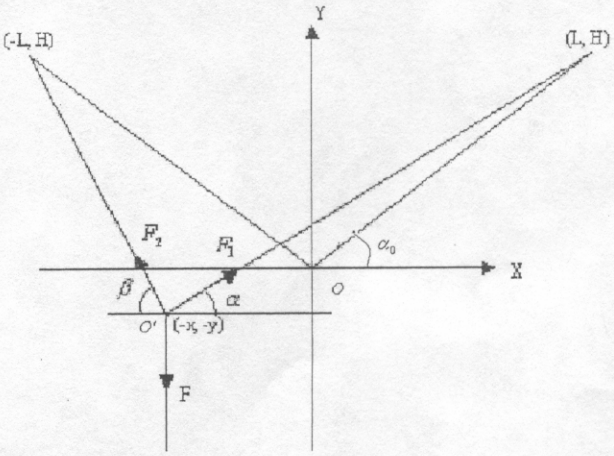

FIGURE 3. Dynamic illustration of a two-strand spun yarn.

Let the ends of the two strands above the convergence point be fixed at a distance $2L$ apart, and the equilibrium position be H. The equations of motion in the x- and y-directions are

$$M \frac{d^2x}{dt^2} + F_1 \cos \alpha - F_2 \cos \beta = 0 \quad , \qquad (1)$$

$$M \frac{d^2y}{dt^2} + F_1 \sin \alpha + F_2 \sin \beta - F = 0 \quad . \qquad (2)$$

Here, M is the total mass of a fixed control volume $ABCD$, illustrated in Figure 4; the control volume is chosen in such a way that the mass center coincides with the convergence point (O) of the two strands. The mass M is then determined from the relation $M = \rho_{1L1} + \rho_{2L2} + \rho h$, where l_1, l_2 and ρ_1, ρ_2 are, respectively, length and density per unit length of two parent strands above the convergence point, h is the distance of the two-strand yarn below the convergence point, and ρ is the density per unit length of the resultant two-strand yarn.

If x and y are much smaller than $\sqrt{L^2+H^2}$, applying the binomial theorem to expand the square-root terms, we have

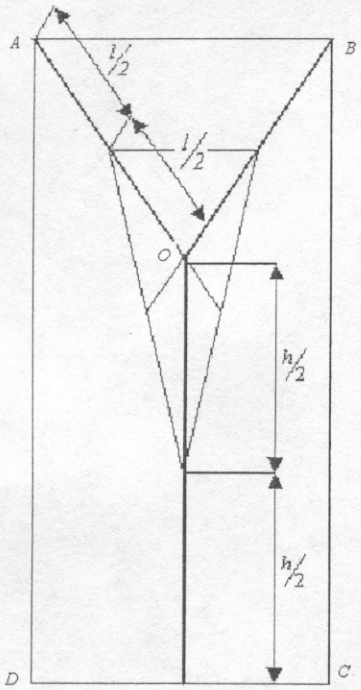

FIGURE 4. Control volume; the mass center at the convergence point.

$$\cos\alpha = \frac{L+x}{\sqrt{(L+x)^2+(H+y)^2}} \approx \frac{L}{\sqrt{L^2+H^2}}\left(1+\frac{x}{L}-\frac{Lx+Hy}{L^2+H^2}\right) \quad , \tag{3}$$

$$\cos\beta = \frac{L-x}{\sqrt{(L-x)^2+(H+y)^2}} \approx \frac{L}{\sqrt{L^2+H^2}}\left(1-\frac{x}{L}-\frac{-Lx+Hy}{L^2+H^2}\right) \quad . \tag{4}$$

Similar expressions can be obtained for $\sin\alpha$ and $\sin\beta$. In this paper when the system is in equilibrium, we consider a simple symmetrical case, *i.e.*, $\alpha_1=\alpha_2$, $F_1 = F_2 = f =$ constant. Equations 1 and 2 can be approximately written in the forms

$$\frac{d^2}{dt^2} + \omega_x^2 x = 0 \quad , \tag{5}$$

$$\frac{d^2y}{dt^2} + \omega_y^2 y = 0 \quad , \tag{6}$$

where ω_x and ω_y are oscillating frequencies in the x- and y-directions, respectively, defined as

$$\omega_x = \sqrt{\frac{fL}{M\sqrt{L^2+H^2}}\left(\frac{2}{L}-\frac{2L}{L^2+H^2}\right)} \quad ,$$

$$\omega_y = \sqrt{\frac{fL}{M\sqrt{L^2+H^2}}\left(\frac{2}{H}-\frac{2H}{L^2+H^2}\right)} \quad .$$

The solutions for x and y can be expressed by

$$x = A\sin\omega_x t \quad , \tag{7}$$

$$y = B\sin\omega_y t \quad , \tag{8}$$

where A and B are amplitudes in the x- and y-directions, respectively. In the case where $\omega_x = \omega_y$ (*i.e.*, $H = L$, $\alpha_0 = \pi/4$), we have

$$y = kx \quad , \tag{9}$$

where $k = B/A$. Equation 9 states that the convergence point (O') always tends to its equilibrium position (O). Various trajectories of the convergence point (O') can be obtained using our quasistatic model [3] by suitable choice of the parameter H, as shown Figure 5.

Conclusions

To conclude, we suggest a linear dynamic model for the two-strand spun yarn process, which can be applied directly to the process investigation. The dynamic character of the system depends strongly on the convergence angles. This study also reveals that the optimal equilibrium convergence angle is $\alpha_0 = \pi/4$, and under such a condition, the system behaves like a mass-spring system. However, the dynamic system, in general, is inherently nonlinear, and we will study the nonlinear case in our next paper.

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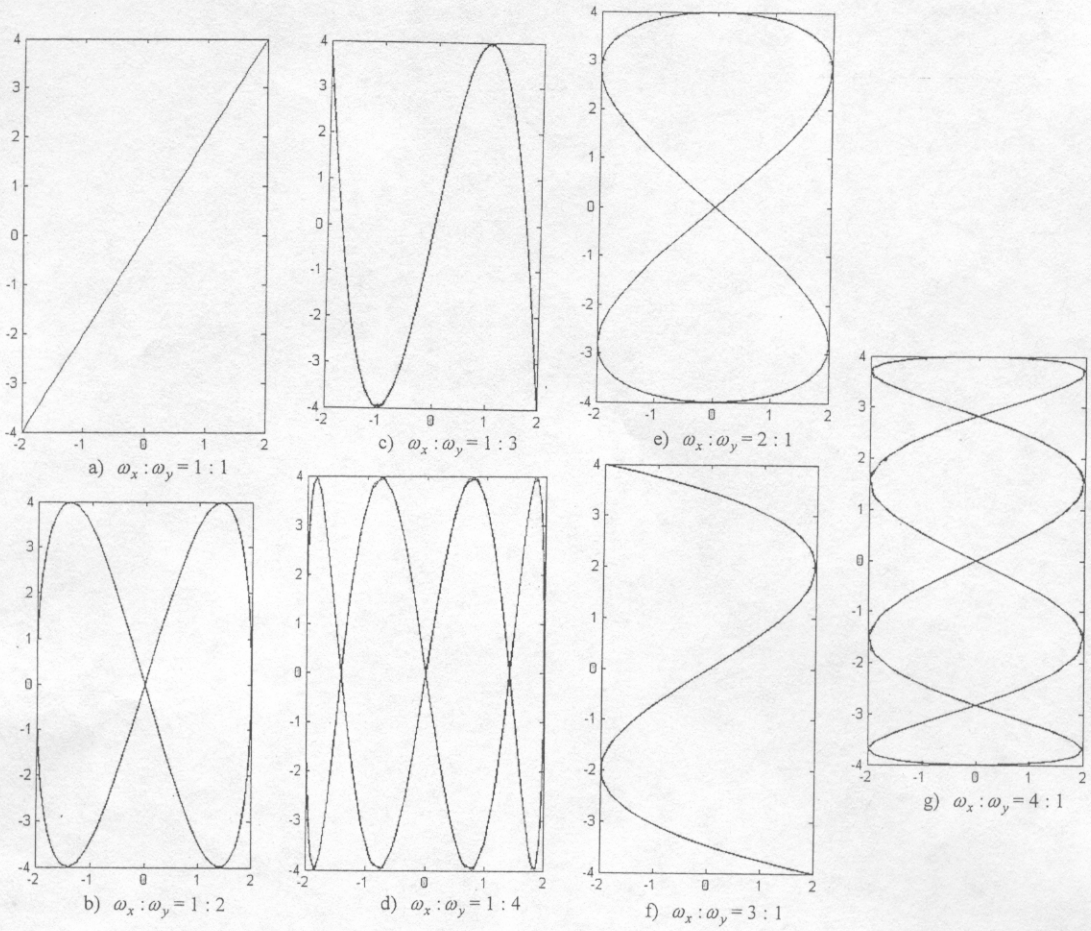

FIGURE 5. Trajectories of the convergence point under different conditions.

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