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Composite Structures 68 (2005) 505–510
COMPOSITE STRUCTURES
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On the Poisson's ratios of a woven fabric

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Available online 15 June 2004

Abstract

Although it is undeniable that the Poisson's effect on the behavior of a woven fabric is crucial, there have been relatively few papers devoted to this subject. In this study, a mechanical model for a woven fabric made of *extensible* yarns is developed to calculate the fabric Poisson's ratios. Theoretical results are compared with the available experimental data. A thorough examination on the influences of various mechanical properties of yarns and structural parameters of fabrics on the Poisson's ratios of a woven fabric is given. The prediction of Poisson's ratios in this paper will enable more rigorous studies on such important issues of fabric bending and draping behaviors.

Keywords: Woven fabrics; Poisson's ratios; Extensible yarns

1. Introduction

The Poisson's ratio is one of the fundamental properties of any engineering materials, and represents important mechanical characteristics for a woven fabric in many applications including in engineering systems which incorporate textile fabrics as structural elements. Such structures include inflatable containers, tires, certain plastic laminated sheets, belting of various kinds, parachutes, sails, mackintoshes, etc. The magnitudes of Poisson's ratios can attain some peculiar values for woven fabrics [1], very different from those for conventional engineering materials, leading to unusual stress–strain relationships.

Literature indicates that many investigators have studied the response of woven fabrics to a planar stress system. Kilby [2] in 1963 put forward a simple trellis model for a woven fabric and analyzed the planar stress–strain relationship, based on the continuum mechanics for an anisotropic elastic lamina; yet the author admitted the deficiency of this theoretical model for not exhibiting the Poisson's ratios. Assuming a relation of yarn curvatures between the released and the stressed states for a woven fabric, Olofsson [3] soon after gave a mathematical analysis of equilibrium conditions, stress–strain relationships in extension and compression, and energy in bending. Grosberg and Kedia [4] analyzed the initial load-extension modulus of a cloth and showed that it depends not only on the bending modulus of the yarn and the geometry it takes up in the cloth, but also on the strain history of the fabric. An excellent summary of the analysis of the mechanical properties of woven fabrics prior to 1969 was included in the monograph by Hearle et al. [5], who derived the Poisson's ratio of a woven fabric assuming that the yarn extension and compression were negligible.

By means of optimal-control theory, de Jong and Postle [1] applied the general energy analysis of fabric mechanics to the woven fabric structure for deformations, where the yarn extension was introduced into the theory. It was noted that the theoretical calculations of the fabric Poisson's ratios based on the assumption of *inextensible* yarns are in conflict with the experimental results. Next, Huang [6] presented a methodology for analyzing the problem of the finite biaxial extension of a plain woven fabric, including such effects as the initial stresses due to partial setting of yarns, loss in bending stiffness associated with fiber slippage in the yarn and the contact deformation of the yarns at the crimp in the analysis. Numerical solutions were found by using the Runge–Kutta method for the problems of fabric uniaxial extension and biaxial extension with equal stresses.

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doi:10.1016/j.compstruct.2004.05.017

But the fabric Poisson's ratios were not dealt with in the analysis. Leaf and Kandil [7] constructed a simple mechanical model termed the 'straight-line' or 'saw-tooth' scheme to represent an idealized woven fabric and presented an analysis of the initial load-extension behavior of plain woven fabrics. A closed-form analytical solution was found for the initial Young's modulus and the Poisson's ratio of the fabric, when the yarns were assumed to be *inextensible* and *incompressible*.

By modeling the individual yarn as *extensible* elastica, Warren [8] in 1990 determined the in-plane linear elastic constants of woven fabrics. Results of this theoretical analysis compare favorably with the measured in-plane Young's moduli of woven fabrics. Pan [9] proposed a fabric model as the chain of yarn sub-bundle. The fabric stress–stain curve and the mean fabric strength were predicted for both uniaxial and biaxial tension cases. Using the finite element method, Tarfaoui et al. [10] recently studied theoretically the mechanical behavior of textile structures of two different weave types: plain and twill. However, they admitted that analyzing their numerical results proved to be very hard and thus demanded a study of the stress field in the fabric unit cell.

Insofar as the experimental determination of fabric Poisson's ratios is concerned, Lloyd and Hearle [11] in 1977 examined their suggested method of a uniaxial tensile test. It was found that the method does not provide a reliable technique and that, to overcome the shortcomings inherent in the specimen geometry, a biaxial test method is needed. Bao et al. [12] in 1997 studied the error sources for measuring the apparent Poisson's ratio of textile fabrics by uniaxial tensile testing. In order to correct the experimental error, samples of various dimensions were utilized for their experiments, and some concrete examples to acquire apparent Poisson's ratios of fabrics by the uniaxial tensile test were also shown.

Up to now, it is hard to get accurate Poisson's ratio measures due to lack of reliable experimental techniques for woven fabrics [13]. While the significance of the effects of Poisson's ratios on fabric drape and other behaviors is well recognized, their values were mostly estimated, based on those for ordinary solid materials, for fabric modeling and simulations [14,15]. Although a few papers dealt with the theoretical prediction of Poisson's ratios for non-wovens [16,17], there has been little done analytically determining Poisson's ratios for woven fabrics based on the assumption of *extensible* yarns in fabrics, and discussing the influences of the mechanical properties of yarns and the geometrical parameters of fabrics on the Poisson's values.

This paper tries to fill the need. First, following the approach put forward by Warren [8], a mechanical model for woven fabrics made of *extensible* yarns is developed to get a closed-form expression for Poisson's ratios. Then, the analytical results are compared with the experimental measurements. Finally, a parametric analysis is given for the effects of various geometric and mechanical parameters on the Poisson's ratios of woven fabrics.

2. Theoretical analysis

A woven fabric is composed of two sets of orthogonal interlaced yarns: warp and weft. The waves in either set of yarns throughout its length are considered as alternating circular arcs [8,18] illustrated in Fig. 1. The structural parameters of pick spacing p, yarn length l and crimp height h can be represented in terms of yarn geometric parameters R and ϕ_0 by

$$p = R\sin\phi_0 \tag{1a}$$

$$l = R\phi_0 \tag{1b}$$

$$h = R(1-\cos\phi_0) \tag{1c}$$

where R is the radius of yarn undulation and ϕ_0 is the crimp angle.

Each yarn is modeled as extensible elastica as shown in Fig. 2. The undeformed shape of the elastica is defined by the slope

$$\phi = \phi(s_0) = \frac{s_0}{R} \quad 0 \leqslant s_0 \leqslant L_0 \tag{2}$$

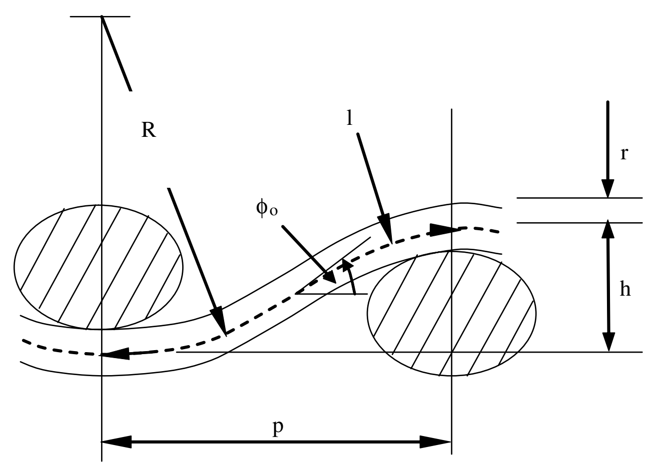

Fig. 1. Geometry of a woven fabric.

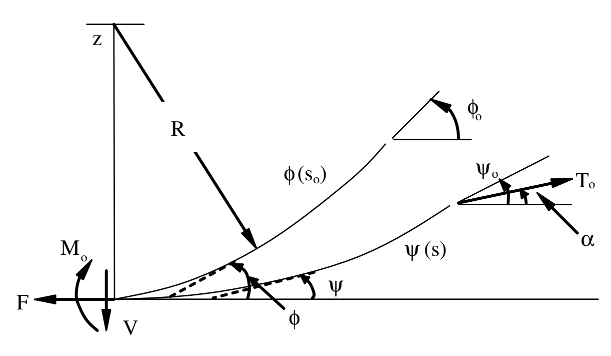

Fig. 2. The *extensible* elastica.

where s_0 is the original arc length along the undeformed curve of total length L_0. The deformed shape is defined by the slope

$$\psi = \psi(s) \quad 0 \leqslant s \leqslant L \tag{3}$$

where s is the arc length along the deformed curve having total length L. The elastica is assumed to stretch linearly under the effect of the axial force $T(s)$ acting through the centroid of the yarn cross-section of area A, which leads to the relation

$$\frac{\mathrm{d}s}{\mathrm{d}s_0} = 1 + \varepsilon(s) = 1 + \frac{\sigma(s)}{E} = 1 + \frac{T(s)}{EA} \tag{4}$$

where E is the Young's modulus of the yarn.

With the force T_0 applied at the end $s = L$ and the reactions F (horizontal) and V (vertical) applied at the symmetry end $s = 0$, the differential equation describing the non-linear deformation of the extensible elastica is [8]

$$EI\frac{\mathrm{d}}{\mathrm{d}s}\left[\left(1 + \frac{T}{EA}\right)\frac{\mathrm{d}\psi}{\mathrm{d}s} - \frac{\mathrm{d}\phi}{\mathrm{d}s_0}\right] = -\frac{\mathrm{d}T}{\mathrm{d}\psi} \tag{5}$$

where I is the moment of inertia for the yarn cross-section. The boundary conditions for the differential equation are

$$\psi(0) = 0 \quad \text{at the symmetry end} \tag{6a}$$

$$\left[\left(1 + \frac{T}{EA}\right)\frac{\mathrm{d}\psi}{\mathrm{d}s}\right]\bigg|_{s=L} = \frac{\mathrm{d}\phi}{\mathrm{d}s_0}\bigg|_{s_0=L_0} \quad \text{at the anti-symmetry end} \tag{6b}$$

Integrating the differential equation (5) twice with the boundary conditions leads to the displacements at the endpoint ($s = L$) of the elastica [8]

$$u_x = \frac{R^3}{4EI}(AF - BV) \tag{7a}$$

$$u_z = \frac{R^3}{4EI}(-BF + CV) \tag{7b}$$

where

$$A = 2(2+\gamma)\phi_0 + 2\phi_0\cos 2\phi_0 - (3-\gamma)\sin 2\phi_0 \tag{8a}$$

$$B = 4\cos\phi_0 - (1+\gamma) - 2\phi_0\sin 2\phi_0 - (3-\gamma)\cos 2\phi_0 \tag{8b}$$

$$C = 2(2+\gamma)\phi_0 - 2\phi_0\cos 2\phi_0 + (3-\gamma)\sin 2\phi_0 - 8\sin\phi_0 \tag{8c}$$

$$\gamma = \frac{I}{AR^2} \tag{8d}$$

In the same way, the displacements at the anti-symmetry endpoint of the other set of yarns orthogonal to the axis x are determined. By applying the equilibrium that requires the contact force V to be the same for both warp and weft yarns, and geometric compatibility that requires the displacements in the vertical direction to be the same, the strain–stress relations are given [8]

$$\varepsilon_{xx} = \frac{R_x^3}{2E_xI_x(\Gamma C_x + C_y)}\left[(\Gamma(A_xC_x - B_x^2) + A_xC_y) \times \left(\frac{R_y\sin\phi_{0y}}{R_x\sin\phi_{0x}}\right)\sigma_{xx} - B_xB_y\sigma_{yy}\right] \tag{9a}$$

$$\varepsilon_{yy} = \frac{R_y^3}{2E_yI_y(\Gamma C_x + C_y)}\left[(\Gamma A_yC_x + (A_yC_y - B_y^2)) \times \left(\frac{R_x\sin\phi_{0x}}{R_y\sin\phi_{0y}}\right)\sigma_{yy} - \Gamma B_xB_y\sigma_{xx}\right] \tag{9b}$$

where the subscripts x and y indicate warp and weft yarns, respectively, and

$$\Gamma = \frac{E_yI_y}{E_xI_x}\left(\frac{R_x}{R_y}\right)^3 \tag{10}$$

The formulae (9a) and (9b) can be written in a matrix form

$$\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\end{bmatrix} = \begin{bmatrix}S_{xx} & S_{xy}\\ S_{yx} & S_{yy}\end{bmatrix}\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\end{bmatrix} \tag{11}$$

It can be readily proved that

$$S_{xy} = S_{yx} \tag{12}$$

Formula (12) is coincident with a requirement for a continuum. Structurally, a woven fabric is not a strict continuum. However, from the point of view of macro-mechanical response to the external force, a woven fabric reacts like a continuum governed by continuum mechanics. Furthermore, Formulae (11) and (12) are in accord with the mechanical constitutive relationship for an orthotropic material with two elastic symmetry axes x (warp) and y (weft).

The Poisson's ratios for a woven fabric are thus determined as follows:

$$\upsilon_{xy} = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}} = \frac{R_x}{R_y}\frac{\sin\phi_{0x}}{\sin\phi_{0y}}\frac{B_xB_y}{\Gamma(A_xC_x - B_x^2) + A_xC_y} \tag{13a}$$

$$\upsilon_{yx} = -\frac{\varepsilon_{xx}}{\varepsilon_{yy}} = \frac{R_y}{R_x}\frac{\sin\phi_{0y}}{\sin\phi_{0x}}\frac{\Gamma B_xB_y}{\Gamma A_yC_x + (A_yC_y - B_y^2)} \tag{13b}$$

From the analysis above, the Poisson's effect in a woven fabric arises from the interaction between the warp and weft yarns, and can be expressed in terms of the structural and mechanical parameters of the system. This is an exclusive characteristic for a fabric, and different from a typical continuum. But their mechanical implications are quite similar. Also note that the effects of the yarn Poisson's ratios themselves are excluded in this analysis.

3. Comparison with experiments and parametric studies

To compare the theoretical results with experiments, the experimental measurements are excerpted from

Table 1
Physical parameters of plain woven fabrics tested [12]

	Material	Mass per unit area (g/m^2)	Thickness (mm)	Picks (number/cm)	
				Warp	Weft
Fabric 1	Polyester	24.3	0.075	53	53
Fabric 2	Polyester	43.2	0.062	106	106
Fabric 3	Cotton	119.6	0.221	27	25
Fabric 4	Wool	136.1	0.240	23	20

Table 2
Theoretical and experimental values of Poisson's ratios of plain woven fabrics

	υ_{xy}		υ_{yx}	
	Theoretical	Experimental [12]	Theoretical	Experimental [12]
Fabric 1	0.473	0.341	0.473	0.341
Fabric 2	0.433	0.202	0.433	0.202
Fabric 3	0.406	0.342	0.489	0.240
Fabric 4	0.380	0.438	0.533	0.567

Ref. [12]. Table 1 shows the specifications of the samples tested. Conferring Fig. 1, the yarn diameter is got by halving the fabric thickness. The comparison of theoretical predictions with the measured results is listed in Table 2. In general, the analytical calculations are in a reasonable agreement with the measurements.

From Formulae (13), (10) and (8), Poisson's ratios for a woven fabric depend on the properties of yarns and structural geometry of fabrics. A parametric study is hence conducted to investigate effects of these factors on Poisson's ratios.

Fig. 3 illustrates the effect of the Young's modulus ratio (E_x/E_y) between warp and weft yarns on the Poisson's ratio υ_{xy} of a woven fabric, for which the weft yarn diameter d_y is 0.04 mm, and the pick spacings are 0.4 mm (25 picks/cm) for both warp and weft yarns. It shows that with the increase of the yarn Young's modulus ratio, the Poisson's ratio increases. With the increase of the diameter ratio (d_x/d_y) between warp and weft yarns, the Poisson's ratio increases as well. The result in Fig. 3 is quite different from the previous conclusions [5,7] based on the assumption of inextensible yarns, where the variation of yarn Young's modulus ratios has no effect on the Poisson's ratio of woven fabrics.

The effect of the yarn diameter ratio (d_x/d_y) on the Poisson's ratio υ_{xy} is demonstrated in Fig. 4 with equal Young's moduli for warp and weft yarns ($E_x/E_y = 1$), while allowing the pick spacing ratio (p_x/p_y) to change from 0.5 to 2.0. It can be seen that with the increase of

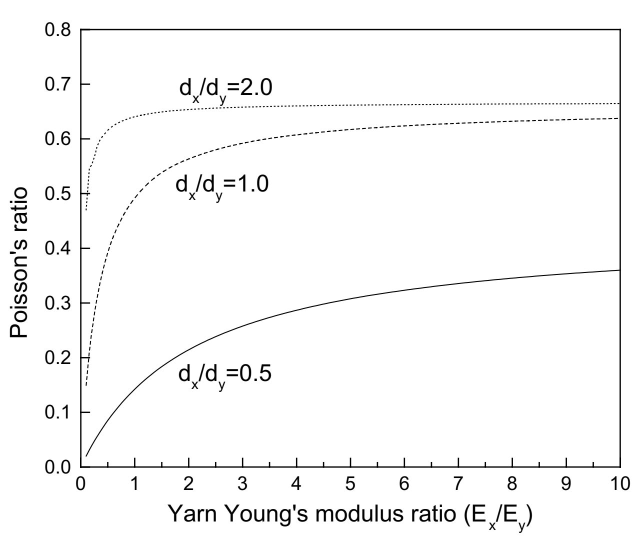

Fig. 3. Variation of Poisson's ratios of woven fabrics with yarn Young's modulus ratios.

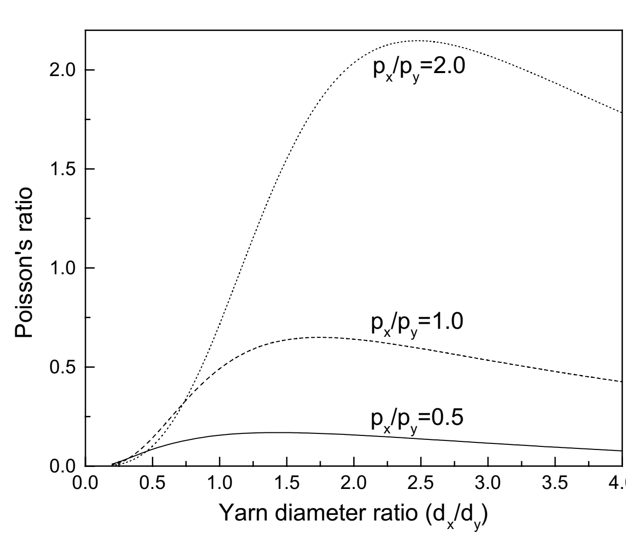

Fig. 4. Variation of Poisson's ratios of woven fabrics with yarn diameter ratios.

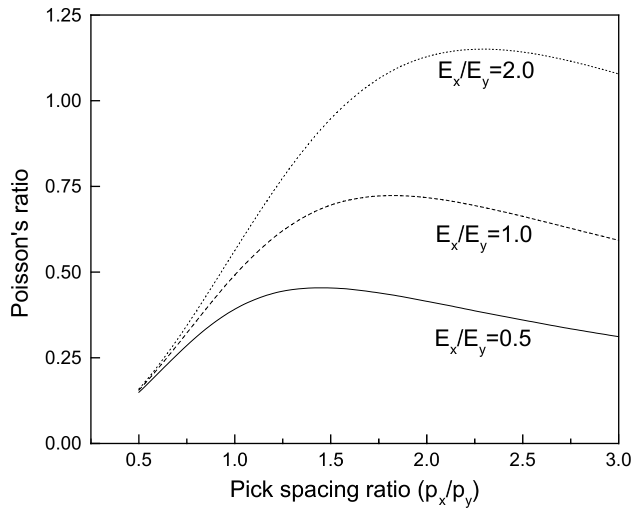

Fig. 5. Variation of Poisson's ratios of woven fabrics with pick spacing ratios.

the yarn diameter ratio, the Poisson's ratio first increases, then decreases after it reaches a maximum. When d_x/d_y is less than 1, the effect of the pick spacing ratio (p_x/p_y) on the Poisson's ratio is complex. While d_x/d_y is greater than 1, increase of the pick spacing ratio boosts significantly the Poisson's ratio, even beyond 2 at some segment of the curve where p_x/p_y equals 2.

Fig. 5 shows the effect of the pick spacing ratio (p_x/p_y) on the Poisson's ratio υ_{xy} of a woven fabric, letting the Young's modulus ratio (E_x/E_y) change from 0.5 to 2.0. With the increase of the pick spacing ratio, the Poisson's ratio first increases, and then decreases after reaching a maximum. Again, the Poisson's ratio increases with the increase of the Young's modulus ratio between the warp and weft yarns.

By comparing Fig. 3 through Fig. 5, it reveals in general that the pick spacing ratio appears to be of the most effect on the Poisson's ratio of a woven fabric, and that the Young's modulus ratio appears to be the least effective. In other words, the impact of structural parameters of a woven fabric on the Poisson's ratio is more significant than that of mechanical parameters.

4. Conclusions

The Poisson's ratio for a woven fabric is predicted by lifting the previous assumption that the yarn in the fabric is inextensible. Theoretical analysis compares favorably with the experimental results.

It is revealed in this study that, the Poisson's effect in a woven fabric arises from the interaction between the warp and weft yarns, and can be expressed in terms of the structural and mechanical parameters of the system; this is an exclusive characteristic for a fabric, and different from a typical continuum. But their mechanical implications are quite similar and the mechanical behavior of a woven fabric can be modeled as a continuum of orthotropy.

The effects of various mechanical properties of yarns and structural parameters of fabrics on the Poisson's ratio of a woven fabric are investigated. On the whole, the pick spacing ratio and the yarn diameter ratio have more important effects on the Poisson's ratio of a woven fabric than the yarn Young's modulus ratio.

This study provides a guideline for the design of a woven fabric.

Acknowledgements

Acknowledgement is due to the financial support from the National Textile Center under the Project F00-MD06.

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