Grip Point Spacing Along the Edges of an Anisotropic Fabric Sheet in a Biaxial Tensile Test

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The technique of gripping fabric edges using pin grips has been applied in several studies of biaxial mechanical behavior of fabrics. This paper presents a theoretical study of the nonuniform stress distribution caused by the spacing of the grip points. The theory leads to expressions of the shear strain in the fabric specimen and the displacement of the grip points by which the tensile stress nonuniformity caused by the grip method can be calculated. The analysis reveals important factors affecting the stress distribution in a fabric specimen and provides design criteria for new instrument development.

INTRODUCTION

Although the applications of woven fabrics have increased, beyond traditional clothing purposes, to industrial and composite areas because of the unique performance of the material, our understanding of the mechanical behavior of woven fabrics is still very limited.

Fabrics are typical porous media and can be treated as mixtures of fibers and air, having no clearly defined boundary and different from a classical continuum. They are not homogeneous nor are they isotropic, i.e., their properties are very susceptible to loading direction. In other words, they behave very differently according to the direction of loading. Therefore, theoretical analysis of fabric behavior becomes very complex, and experimental verification of theoretical predictions is more critical than for other materials.

The tensile test is the most frequently employed experimental approach for the study of fabrics, including both uniaxial and biaxial extension. During tensile tests, a fabric is stretched by gripping the specimen edges using one of several methods.

Treloar (1) was perhaps the first to employ pin grips in order to apply forces to a square sheet specimen as illustrated in *Fig. 1a*. This method was later adopted and modified by other researchers (2–4) and has recently been used to study the stability of fabrics under biaxial stretching (5).

It is most desirable that the distribution of stress and strain in the specimen be homogeneous, so that one can infer the accurate stress-strain relationship from the test results for the material. Unfortunately, the stress distribution in the specimen is subject to error because of the inhomogeneity of stress induced by the clamps. This paper presents a theoretical study of the nonuniform stress distribution caused by the spacing of the grip points. The theory reveals important factors affecting the stress distribution in a fabric specimen and provides some design criteria.

A new biaxial fabric tester developed by authors based on the new theory will be introduced in a separate paper. This theory will also be useful when dealing with other types of biaxial fabric testers such as the grab specimen (6, 7) and the cruciform specimen (8–11) using solid clamps, and also the technique using segmented clamps (12–15) (see *Fig. 1*), as well as uniaxial fabric tensile testers.

THEORY

The following analysis should indicate the order of magnitude of the effect of grip spacing on the stress

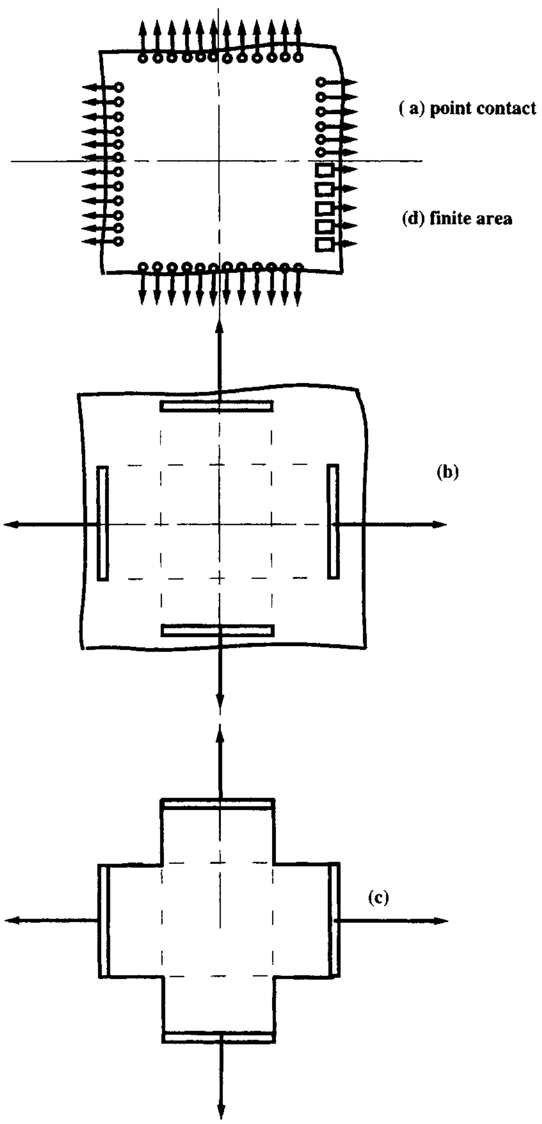


Fig. 1. Different types of biaxial fabric testers: (a) Point gripping; (b) Grab test; (c) Cruciform test; (d) Segmented clamps.

distribution in fabric testers that grip their specimen with point grips. A more accurate result could be obtained, for example, using a finite element technique.

Figure 2a is a schematic diagram of a specimen that is under a load regularly distributed along the specimen edges. The mean value of the uniaxial stress, σ_x, equals the sum of the forces divided by the specimen width. The coordinate system in x refers to the unstrained state of the specimen. The stress σ_x is applied at discrete points, spaced $2h$ apart. The tensile stress in the horizontal x direction at points along section AA′ will, by intuitive reasoning, be small at the edges of the specimen and will approach the average stress, σ_x, towards the center.

The specimen is divided into horizontal strip elements of width h, and the assumption is made that all tensile forces act along the horizontal edges of these elements. A further assumption is that yarns, which

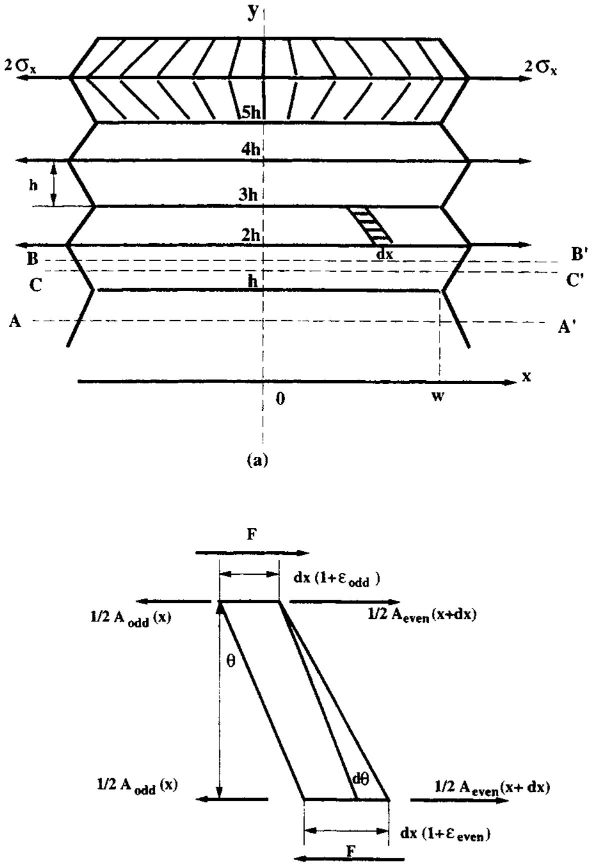


Fig. 2. A stretched fabric specimen: (a) A fabric specimen gripped at discrete points along opposite edges; (b) A yarn segment from the stretched specimen.

were initially straight in the y direction, deform into the herringbone pattern of straight segments shown in the *Figure*.

Now consider narrow strips, which include the section AA′, delimited by pairs of section lines such as BB′, and CC′. In a continuum, the only way in which tensile stress can be applied to these strips is by reaction to the shear stress along the section lines (BB′ and CC′) at the edges of the strips.

Consider the piece of the specimen around the line $j = 1$ in *Fig. 2a*, isolated by section line AA′, and BB′. Tensile stress along $j = 1$ is generated by:

1. shear forces due to shear in the herringbone elements.
2. the horizontal components of the tensions in the inclined directions of the herringbone elements.

The latter effect is neglected in this analysis. The effect of this assumption will be that the predicted stress distribution is more uneven than it would be otherwise.

At the edges ($x = w$), the stress in the even (i.e., j *even*) lines is $2\sigma_x$ and in the odd lines it is zero. As one moves toward the center, stress is transferred from the *evens* to the *odds*, such that the sum is always $2\sigma_x$.

The shear strain, θ, varies from zero at $x = 0$ (by symmetry), to some positive value at the edge, owing to the cumulative difference in tensile strain between the *odds* and *evens*. *Figure 2b* illustrates an element and its increase, $d\theta$ in θ. It is assumed that θ is small, so that $\cos\theta \approx 1$. It follows that:

$$d\theta = dx \frac{(\varepsilon_{even} - \varepsilon_{odd})}{h} \quad (1)$$

It is possible that θ in a real fabric will become large, so that fabric jamming may occur, which would help the stress transfer. Additionally, the tension in the element in the y-direction would then have significant effects.

The value of the forces F, due to the shear of the element in *Fig. 2b*, is:

$$F = \tau dx \quad (2)$$

where τ is the shear stress. Assume that the material has linear stress-strain properties, which is largely true for the biaxial tensile case (5), with Young's modulus E_x in the x-direction, and shear modulus G_{xy}. Effects in the y-direction, including Poisson's effects, are neglected. Then from *Eq 2*,

$$F = G_{xy}\theta dx \quad (3)$$

Let the axial forces in the horizontal lines be given by some functions $A_{odd}(x)$ and $A_{even}(x)$. The equations of equilibrium in the x-direction of the ends of the element in *Fig. 2b* are:

odd lines:

$$\frac{1}{2} A_{odd}(x + dx) = \frac{1}{2} A_{odd}(x) - F \quad (4)$$

The factor $\frac{1}{2}$ is because there are elements both above and below the line.

even lines:

$$\frac{1}{2} A_{even}(x + dx) = \frac{1}{2} A_{even}(x) + F \quad (5)$$

i.e.:

$$A_{odd}(x + dx) - A_{odd}(x) = -2G_{xy}\theta dx \quad (6)$$

or

$$dA_{odd} = -2G_{xy}\theta dx \quad (7)$$

Integrating,

$$A_{odd} = -2G_{xy}\int \theta dx \quad (8)$$

A_{odd} represents the tensile force in a strip of width h, so that the tensile stress is:

$$\sigma_{odd} = -\frac{2G_{xy}}{h}\int \theta dx \quad (9)$$

Applying Hooke's Law,

$$\varepsilon_{odd} = -\frac{2G_{xy}}{hE_x}\int \theta dx \quad (10)$$

Similarly,

$$\varepsilon_{even} = \frac{2G_{xy}}{hE_x}\int \theta dx \quad (11)$$

Substituting into *Eq 1*,

$$d\theta = dx \frac{4G_{xy}}{h^2E_x}\int \theta dx \quad (12)$$

i.e.:

$$\frac{d\theta}{dx} = \frac{4G_{xy}}{h^2E_x}\int \theta dx \quad (13)$$

or, differentiating,

$$\frac{d^2\theta}{dx^2} = \frac{4G_{xy}}{h^2E_x}\theta \quad (14)$$

Putting $k = \frac{4G_{xy}}{h^2E_x}$, the solution to this equation can be found using standard methods:

$$\theta = c_1 e^{\sqrt{k}x} + c_2 e^{-\sqrt{k}x} \quad (15)$$

At $x = 0$, $\theta = 0$, whence $c_1 + c_2 = 0$. i.e.,

$$\theta = c_1(e^{\sqrt{k}x} - e^{-\sqrt{k}x}) = 2c_1 \sinh(\sqrt{k}x) \quad (16)$$

and, differentiating *Eq 16* and equating it to *Eq 1* yields

$$c_1 = \frac{\sigma_x}{hE_x\sqrt{k}\cosh(\sqrt{k}w)} \quad (17)$$

Note that at $x = w$, $\sigma_{even} = 2\sigma_x$, and $\sigma_{odd} = 0$.

Substituting *Eq 15* into *Eq 9* and integrating,

$$\sigma_{odd} = -\frac{4G_{xy}c_1}{h\sqrt{k}}[\cosh(\sqrt{k}x) + c_3] \quad (18)$$

At $x = w$, $\sigma_{odd} = 0$, whence:

$$c_3 = -\cosh(\sqrt{k}w) \quad (19)$$

Substituting in the values of the constants, and canceling common terms:

$$\theta = \frac{\sigma_x \sinh\left(\frac{2x}{h}\sqrt{\frac{G_{xy}}{E_x}}\right)}{\sqrt{E_xG_{xy}}\cosh\left(\frac{2w}{h}\sqrt{\frac{G_{xy}}{E_x}}\right)} \quad (20)$$

The shear stress follows:

$$\tau = G_{xy}\theta = \sqrt{\frac{G_{xy}}{E_x}}\frac{\sigma_x \sinh\left(\frac{2x}{h}\sqrt{\frac{G_{xy}}{E_x}}\right)}{\cosh\left(\frac{2w}{h}\sqrt{\frac{G_{xy}}{E_x}}\right)} \quad (21)$$

and the tensile stress

$$\sigma_{odd} = \sigma_x \left[1 - \frac{\cosh\left(\frac{2x}{h}\sqrt{\frac{G_{xy}}{E_x}}\right)}{\cosh\left(\frac{2w}{h}\sqrt{\frac{G_{xy}}{E_x}}\right)} \right] \quad (22)$$

σ_{even} can be found using $\sigma_{even} = 2\sigma_x - \sigma_{odd}$.

The total displacement of the grip points is given by:

$$g = \frac{2}{E_x}\int_0^w \sigma_{even} dx = \frac{2w\sigma_x}{E_x} \left[1 + \frac{h}{2w\sqrt{\frac{G_{xy}}{E_x}}} \tanh\left(\frac{2w}{h}\sqrt{\frac{G_{xy}}{E_x}}\right) \right] \quad (23)$$

where the factor 2 accounts for the total sample length of $2w$.

The average tensile strain ε is obtained from

$$\varepsilon = \frac{g}{2w} = \frac{\sigma_x}{E_x}\left[1 + \frac{h}{2w\sqrt{\frac{G_{xy}}{E_x}}} \tanh\left(\frac{2w}{h}\sqrt{\frac{G_{xy}}{E_x}}\right) \right] \quad (24)$$

It is thus clear that the term containing *tanh* is the difference between the results with discrete grip-points and those with homogeneous strain, and this term approaches zero as h approaches zero.

3 PREDICTIONS AND DISCUSSION

Details on the fabric specimen and specifications used in this study are provided in *Table 1*. For a complete description of the specimen performance, various measured properties are shown in the *Table*, although not all data are used in the calculations.

Table 1. Specifications of Fabrics.

Material	70% wool, 30% polyester
Weave	Plain
Yarn counts	52 tex (weft & warp)
Sample length (2w)	100 mm
Warp density	1730/meter
Weft density	1970/meter
Bending Stiffnesses	
B_x (warp)	6.93×10^{-9} *Nm²/thread*
B_y (weft)	1.20×10^{-5} *Nm²/thread*
Young's moduli	
E_x (warp)	6450 Nm^{-1}
E_y (weft)	2000 Nm^{-1}
Shear modulus G_{xy}	30.1 Nm^{-1}
Frictional shear restraint σ_o	0.73 Nm^{-1}
Poisson's ratio	
ν_{xy}	0.41
ν_{yx}	0.42

3.1 The Shear Stress Distribution in the Specimen

The shear stress at the edge point x of the specimen is given in Eq 21 as a function of $\sqrt{\frac{G_{xy}}{E_x}}$, the sample dimension w and the spacing between grips h at a given external tensile stress σ_x as also shown in *Figs. 3a, b,* and *c*. Note that all these figures are plotted relative to the original tensile stress σ_x applied.

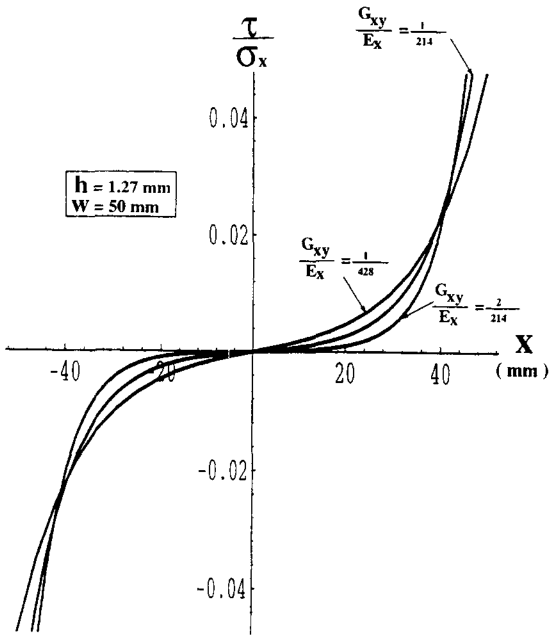
Figure 3a illustrates the distribution of the shear stress along the fabric sample length and the effect of the moduli ratio $\sqrt{\frac{G_{xy}}{E_x}}$, while the parameters h and w remain constant. The shear stress is the maximum at the both ends of the sample, and descends inward until the center where the shear stress diminishes. A fabric sample with a higher $\sqrt{\frac{G_{xy}}{E_x}}$ value, usually meaning a tighter fabric structure or higher inter-yarn friction, will have a steeper descending process.

The effects of the spacing h and sample size w at given $\sqrt{\frac{G_{xy}}{E_x}}$ ratio can be seen in *Figs. 3b* and *c*, respectively. It shows that a decrease in spacing h has the similar influence on the shear stress as an increase in sample size w; both lead to a more rapid descending in the shear stress.

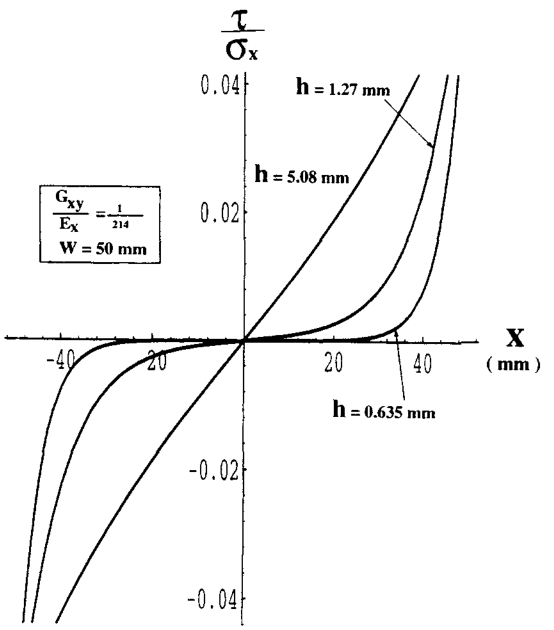
It is also seen in *Eq 21* that the shear stress τ acquires the maximum value τ_{max} at the both edges of the sample where $x = w$, and this maximum value is determined by the moduli ratio $\sqrt{\frac{G_{xy}}{E_x}}$ and the ratio $\frac{w}{h}$, with the former having a dominant effect as depicted in *Fig. 4*. The τ_{max} value increases monotonically with $\sqrt{\frac{G_{xy}}{E_x}}$. At the range where $\sqrt{\frac{G_{xy}}{E_x}}$ is still small, the ratio $\frac{w}{h}$, has some influence on the τ_{max} value and a greater $\frac{w}{h}$ yields a higher τ_{max}. Once $\sqrt{\frac{G_{xy}}{E_x}}$ is beyond a certain value, the effect of $\frac{w}{h}$ diminishes. Note that a greater τ_{max} value means a more effective tensile stress transferring, a more rapid shear stress descending, and therefore a less complex stress field in the fabric sample.

3.2 The Tensile Stress Distribution in the Specimen

The tensile stress is built up with the assistance of shear stress. The effects of important parameters on its distribution can be seen from *Figs. 5a, b,* and *c* for the tensile stress σ_{odd} in *Eq 22* in relative scale.



(a)



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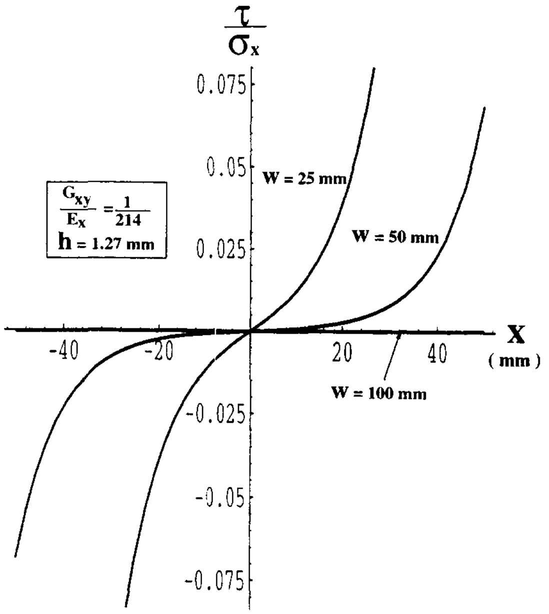


(c)

Fig. 3. Shear stress distribution on a stretched fabric specimen: (a) The shear stress distribution and the influence of the moduli ratio $\sqrt{\frac{G_{xy}}{E_x}}$*; (b) the shear stress distribution and the influence of the spacing of the grip points h; (c) The shear stress distribution and the influence of sample size w.*

It is seen in the *Figures* that the tensile stress is zero at both ends of the sample and ascends to the maximum value at the center of the sample. If the related parameters are in the proper range, this tensile stress can reach the level of the original tensile stress σ_x applied. It can also be concluded from the *Figures* that a tighter fabric with a greater moduli ratio $\sqrt{\frac{G_{xy}}{E_x}}$, or a small spacing h or a larger sample size w, will generate a more uniform tensile stress distribution along the sample length. In other words, if all other parameters are fixed, a small h value, i.e., more gripping points, leads to a more uniform distribution of tensile stress along the cross section AA′ in *Fig. 2a*.

3.3 The Displacement of the Grip Points and the Error Analysis

It is clear in *Eq 24* that the error e, the term containing *tanh*, caused by the nonuniformity of the tensile stress distribution is related to the two ratios $\sqrt{\frac{G_{xy}}{E_x}}$ and $\frac{w}{h}$ as illustrated in *Fig. 6*.

There will be less error in a tighter fabric, or a fabric with higher internal friction, i.e., a greater $\sqrt{\frac{G_{xy}}{E_x}}$.

When fabrics are given so that $\sqrt{\frac{G_{xy}}{E_x}}$ is constant, the nonuniformity in the sample can be reduced by either narrowing the spacing between the grip points or by choosing a longer sample.

The detrimental effect of the variation in stress across the specimen is that it increases the displacements of the grip-points, which increases the estimate of strain calculated from these displacements. The total displacement of the grip points can be calculated from *Eq 23*, and the effects of the important parameters on it can be deduced from *Fig. 6* as well.

Furthermore, some specific values of the errors in the estimate of strain, which would result from these effects, and values of the shear strain at the edges, are set out in *Table 2* by using the parameters from the actual fabric sample in *Table 1*. The analysis in *Table 2* confirms that the spacing of the grip-points on some previous biaxial testers was too wide (2), and suggests that a spacing of 1.27 mm (h = 0.635 *mm*) is closer to the ideal.

We have noticed by actually examining the stretched specimens that the shear strain between the grip-points does not approach the predicted value of 0.7 radian. While this could be explained by the occurrence of fabric jamming, there is another effect that can be large at small grip-point spacing, viz, the effect of the resistance to bending of the yarns that are parallel to the gripped edge.

The order of magnitude of values of h for which this yarn bending effect is significant, can be estimated, using classical beam theory. *Figure 7a* represents the fabric near the grip-points, and *Fig. 7b* shows a beam that is equivalent to one of the yarns parallel to the gripped edge.

Classical beam theory (e.g., 16) shows that, under some force F, the deflection, D, is given by:

$$D = \frac{Fh^3}{24B} \tag{25}$$

where B is the bending rigidity of the yarn. The deflection due to shear alone would be $\frac{Fh}{G_t}$, where G_t is the fabric shear modulus per thread. The bending effect will be comparable to shear deformation if:

$$\frac{Fh}{G_t} = \frac{Fh^3}{24B} \tag{26}$$

i.e. if:

$$h = 2\sqrt{\frac{6B}{G_t}} \tag{27}$$

For the experimental fabric, the value of this expression can be calculated by using the data in *Table 1* where $G_t = G_{xy} = 30.1\ \frac{N}{m}$, $B = B_x = 6.93 \times 10^{-9}\ N\ M^2/thread = 6.93 \times 10^{-9}\ N\ m^2 \times 1730\ m^{-1} = 11.99 \times 10^{-6}\ N\ m$. So the value h = 3.09 *mm*, close to the grip spacing, which indicates that the tensile stress distribution in this fabric will be more uniform than *Fig. 5* would predict.

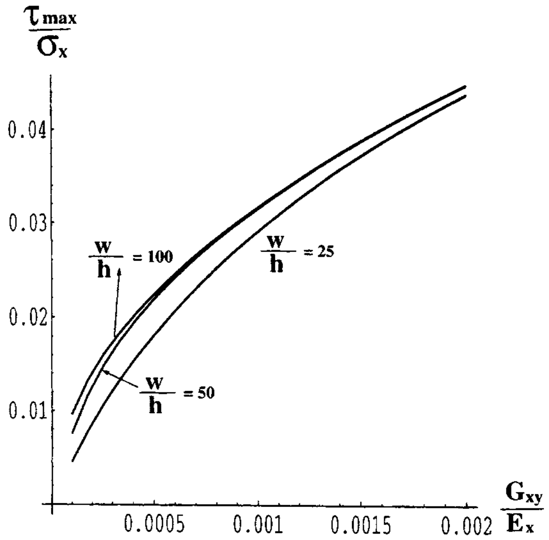
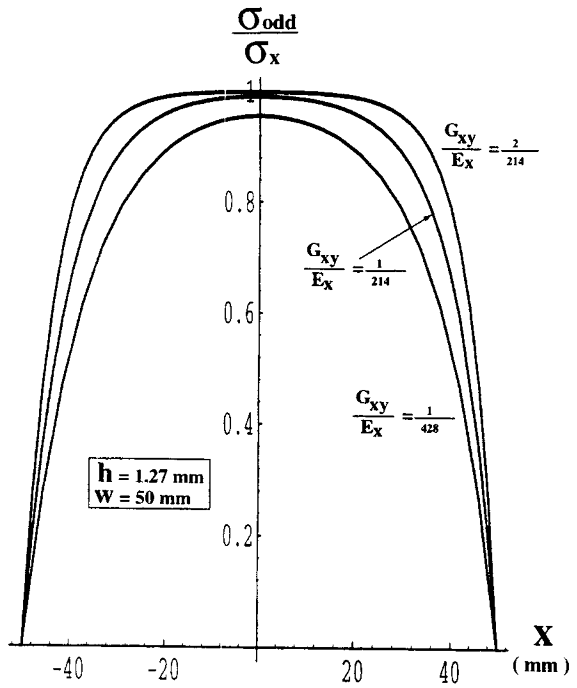


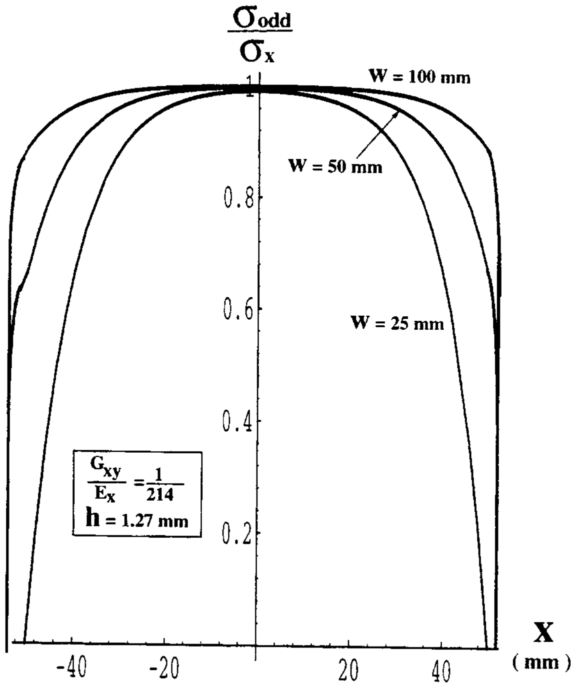
Fig. 4. The relative maximum shear stress $\frac{\tau_{max}}{\sigma_x}$ *and the related parameters.*

Table 2. Calculated Effects of Grip-Point Spacing $\left(\sqrt{\frac{G_{xy}}{E_x}} = 214,\ w = 50\ \text{mm}\right)$.

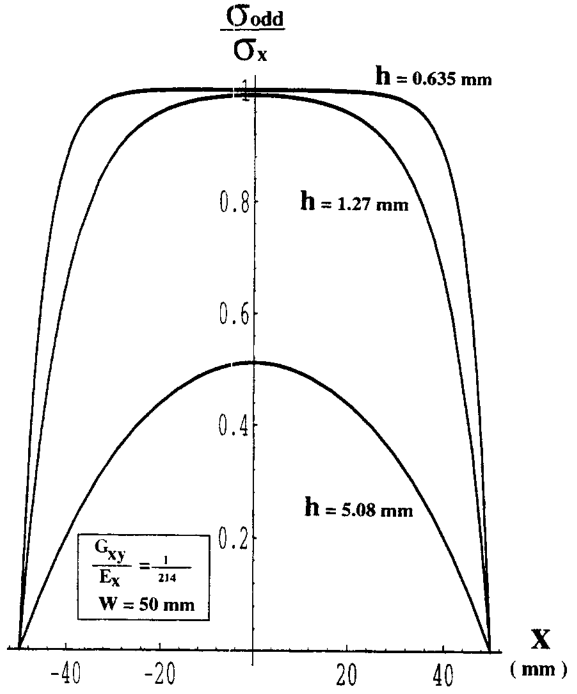
Grip-point spacing (2h)	1.270 mm (20 wires/in.)	2.54 mm (10 wires/in.)	10mm (2.5 wires/in.)
error in strain in x-direction, as would be estimated from the edge displacements	9%	19%	64%
shear strain at edge σ_x = 350*N/m*	0.70 rad	0.70 rad	0.70 rad



(a)



(b)



(c)

Fig. 5. Tensile stress distribution on a stretched fabric specimen: (a) The tensile stress distribution and the influence of the moduli ratio $\sqrt{\frac{G_{xy}}{E_x}}$*; (b) the tensile stress distribution and the influence of the spacing of the grip points h; (c) The tensile stress distribution and the influence of sample size w.*

This analysis of yarn bending rigidity effects is not directly applicable to knitted fabrics, but the stress distribution should be better in these fabrics, because their $\sqrt{\frac{G_{xy}}{E_x}}$ ratios are higher than for woven fabrics.

If the woven fabric is mounted with its weft yarns at some angle α_0 to the row of grips, then analyses of the two sets of yarns would be similar to the procedure described above, but the expression $h \sin \alpha_0$ should be substituted for h when analyzing the weft, and $h \cos \alpha_0$ should be used for the warp, i.e., the stress distribution should be more uniform in woven fabrics mounted at bias angles.

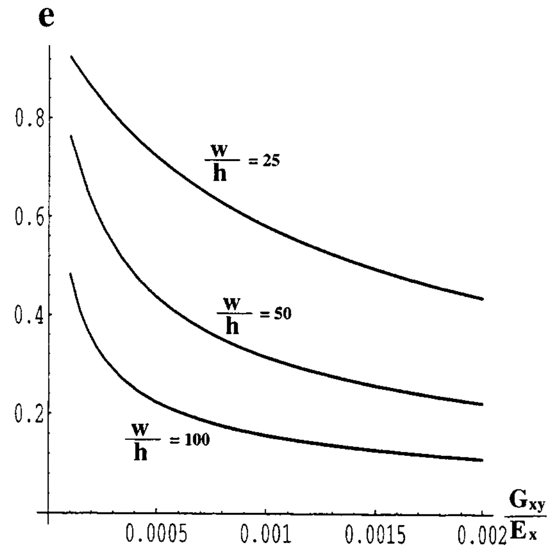


Fig. 6. The error analysis.

4 CONCLUSIONS

Theoretical analyses of the stress nonuniformity and errors in the fabric point-gripping technique lead to the following conclusions:

1. The shear and tensile stresses distributions caused by the grips are nonlinear. The shear stress is the maximum at the edges of the fabric and reduces gradually to zero toward the center, and the tensile stress is generated via the shear stress and therefore is zero at the fabric edges and reaches the maximum at the fabric center. This maximum tensile stress value can be as high as the original tensile stress σ_x applied if all the related parameters are at the proper levels.
2. The shear and tensile stresses, as well as the total displacement of the grip points, are all the functions of the moduli ratio $\sqrt{\frac{G_{xy}}{E_x}}$, the spacing of grip points h and the sample length $2w$. If all other parameters are given, one can improve the uniformity of the stress distributions and reduce the error caused by the grip points either by choosing a tighter fabric or a fabric with higher inter-yarn friction, or by reducing the spacing of grip points or by increasing the sample length.
3. Although the maximum tensile stress is a constant equal to the original tensile stress σ_x applied, the maximum shear stress τ_{max} is shown to be determined mainly by the moduli ratio $\sqrt{\frac{G_{xy}}{E_x}}$, and the ratio $\sqrt{\frac{w}{h}}$ only has a marginal effect when ratio $\sqrt{\frac{G_{xy}}{E_x}}$ is small.
4. The effect of bending stiffness of the yarns in a stretched fabric sample can alleviate the error due to grip points.
5. Because of their low shear moduli, the proper gripping of woven fabrics requires the grip points to be spaced not further apart than a critical h

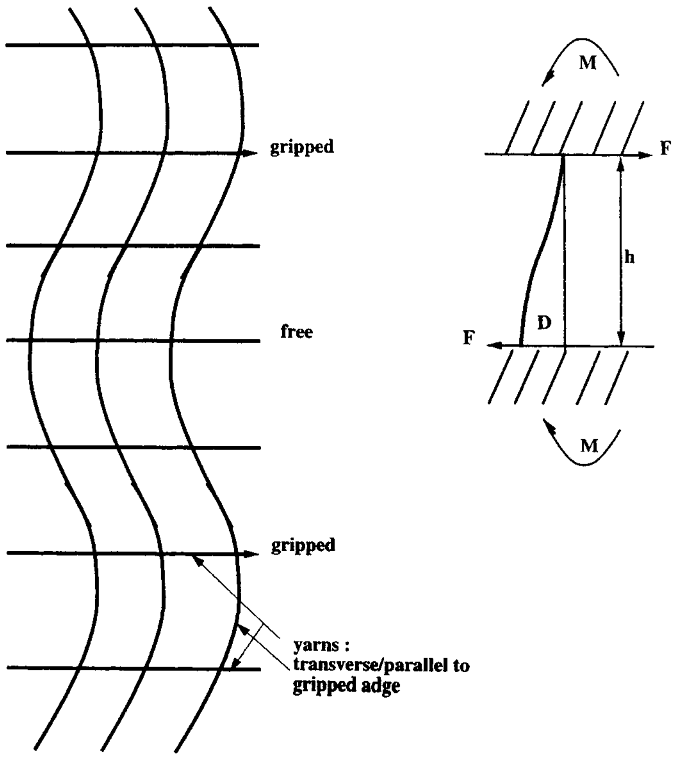


Fig. 7. Effects of yarn bending rigidity: (a) The woven fabric specimen; (b) Equivalent yarn beam.

value, for the fabric in *Table 1* where sample length $2w$ = 100, this value h = 1.3 *mm* (or 20 grip points per inch). Grip point spacing is less critical when woven specimens are mounted at bias angles.

ACKNOWLEDGMENT

Ning Pan would like to express his gratitude for a grant from the Australian Research Council, which enabled him to spend two months visiting the University of New South Wales and join this ongoing research project.

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