Investigation on the strength–size relationship in fibrous structures including composites

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The size effects due to changes in gauge length and the influence of the fragmentation phenomenon in fibrous structures are examined. First, a theoretical analysis of the differences of the size effects in single fibre and in a fibrous structure is conducted. Then comprehensive experimental work is presented on single fibres, fibre bundles, and twisted yarns which can be considered as pseudo-composites. Next a comparison is made between the theoretical predictions and the experimental data. Causes for the size effect in a fibrous structure are explored.

1. Introduction

In many practical applications, especially in the aircraft and civil engineering industries, full scale testing is very costly or sometimes impossible to conduct. Hence, it is an extremely useful pursuit to find ways by which such testing can be reduced in scale without the loss of original physics. Establishment of the connections between two physical systems differing only in scale thus becomes highly desirable.

The size or length effect in single fibres has been well recognized and thoroughly studied. This effect is observed not only with flaw-sensitive brittle fibres [6], but also ductile polymer fibres [19]. Furthermore, it is now widely accepted that the fibre strength–length relationship can be described by the Weibull statistical model [26].

For fibrous structures such as textiles, paper and fibre-reinforced composites, the issue becomes a little more complex. The size effect in textile yarns was first studied by Peirce [20] who proposed the "weakest link" theory to characterize it. The size effect on composite strength has also grown into a very active area for research with numerous studies being published [1–3, 6–8, 11, 16, 21–23, 27, 28]. Yet, the existence of this effect in fibrous structure seems to have become less certain after some recent theoretical investigations brought ambiguity into the problem. It has become well known that, in a fibrous structure under extension, fragmentation occurs prior to the failure of the structure. As a result, the fibres will eventually break into much shorter lengths, better known as the critical length l_c, before complete system failure. Therefore, the new theories on the strengths of composites [8, 24] or textile yarns [18] have predicted that the ultimate strength of a fibrous structure should be calculated by scaling the structure length down to this critical length. The strength thus predicted is closer to the actual value, and much higher than the strength calculated based on the original structure length l_f. However, according to these theories, the strength of a fibrous structure would be, as claimed in [7], independent of the structure length or size, determined chiefly by l_c which is generally not related to l_f. However, this conclusion is in conflict with some of the experimental findings, for example [20, 28]. Zweben [28] has listed many results as evidence for a size effect. Also he has explored the reasons why the size effect in composites has not been widely recognized and the implications of the size effect for composite applications.

It is the purpose of this study to investigate the issue of size effect on the strengths of fibrous structures. Through both theoretical and experimental approaches, we will examine the existence of the size effect in fibrous structure, and explain the differences of size effect on the tensile strengths of fibres, fibre bundles and twisted fibre bundles or yarns.

Previously Pan has pointed out the similarities between a fibre-reinforced composite and a twisted yarn [18]. Mechanistically, a twisted yarn can be treated as a composite where individual fibres are embedded in the matrix formed by neighbouring fibres. The only major difference between the two structures lies in the nature of the fibre–matrix bonding. In a composite, bonding is largely chemical whereas in a yarn is entirely frictional. Therefore, although only yarn samples were used in this study, the conclusions we draw from this study are valid for fibre-reinforced composites as well.

2. Size effect on fibre and bundle strengths

It has been widely accepted that for brittle fibres such as glass, ceramic, carbon, and some polymer fibres [19], the strength cummulative probability distribution obeys the Weibull function. Namely, for a fibre of length l_f, the probability of the fibre strength being σ_f is given by

$$F(\sigma_f) = 1 - \exp[-l_f \alpha \sigma_f^{\beta}] \tag{1}$$

where α is the scale parameter and β is the shape parameter of the fibre and both are independent of the fibre length l_f. The shape parameter β is an indicator of the fibre strength variation. A higher β value corresponds to a lower variance, and when $\beta \to \infty$, the fibre variation would approach zero and its strength would become independent of its length.

The mean or the expected value of the fibre strength $\bar{\sigma}_f$ can then be calculated as

$$\bar{\sigma}_f = (l_f \alpha)^{-1/\beta} \Gamma\left(1 + \frac{1}{\beta}\right) \tag{2}$$

where $\Gamma()$ is the Gamma function, and the standard deviation of the strength is

$$\Theta_f = \bar{\sigma}_f \left[\frac{\Gamma\left(1 + \frac{2}{\beta}\right)}{\Gamma^2\left(1 + \frac{1}{\beta}\right)} - 1 \right]^{1/2} \tag{3}$$

Note that, as pointed out in [25], Equation 1 may not always be accurate for some fibres; it may overestimate the strength for shorter fibre lengths while underestimating it for longer lengths. However, we can state that the present analysis focuses only on fibres whose strength distributions are strictly Weibull forms. In addition, we have tested validity of Weibull function for the three types of polymeric fibres used in this study, and the results are satisfactory [19].

Then according to Daniels in [9], for a fibrous system where N fibres form a parallel bundle with no interaction between individual fibres, the density distribution function of the bundle strength σ_p approaches a normal form

$$H(\sigma_p) = \frac{1}{(2\pi)^{1/2}\Theta_p} \exp\left[-\frac{(\sigma_p - \bar{\sigma}_p)^2}{2\Theta_p^2} \right] \tag{4}$$

where $\bar{\sigma}_p$ is the expected value of the bundle strength

$$\bar{\sigma}_p = (l_f \alpha \beta)^{-1/\beta} \exp\left(-\frac{1}{\beta}\right) \tag{5}$$

and Θ_p is the standard deviation of the strength

$$\Theta_p^2 = (l_f \alpha \beta)^{-2/\beta} \left[\exp\left(-\frac{1}{\beta}\right) \right] \left[1 - \exp\left(-\frac{1}{\beta}\right) \right] N^{-1} \tag{6}$$

It is well recognized that the expected strength of a fibre bundle is lower than that of the fibre by a factor Φ, which is the ratio of Equations 2 and 5, and is sometimes called the "Coleman factor" [4]

$$\Phi = \frac{\bar{\sigma}_f}{\bar{\sigma}_p} = (\beta)^{1/\beta} \exp\left(\frac{1}{\beta}\right) \Gamma\left(1 + \frac{1}{\beta}\right) \tag{7}$$

The strength variation of the fibre bundle is also smaller, depending among other factors on N, than that of the fibre given in Equation 3.

The size effect can be better specified by the derivative of strength with respect to length. Then for a single fibre

$$\frac{d\bar{\sigma}_f}{dl_f} = \frac{-\alpha(l_f \alpha)^{-1-1/\beta} \Gamma\left(1 + \frac{1}{\beta}\right)}{\beta} \tag{8}$$

and for a fibre bundle

$$\frac{d\bar{\sigma}_p}{dl_f} = \frac{-\alpha(l_f \beta \alpha)^{-1-1/\beta}}{\exp\left(-\frac{1}{\beta}\right)} \tag{9}$$

The ratio of the two gives, somewhat surprisingly, the same result as in Equation 7

$$\Phi = \frac{d\bar{\sigma}_f}{d\bar{\sigma}_p} = (\beta)^{1/\beta} \exp\left(\frac{1}{\beta}\right) \Gamma\left(1 + \frac{1}{\beta}\right) \tag{10}$$

In other words, the "Coleman factor" Φ is both a strength ratio and a size effect ratio between the fibre and fibre bundle.

So several interesting points can be made about the "Coleman factor":

1. Φ specifies the translation efficiency from fibre strength into bundle strength. A higher Φ value indicates a greater discrepancy between fibre and bundle strengths, i.e. a lower translation efficiency of fibre strength into bundle strength.
2. Φ also specifies difference in sensitivity of the size effect on the strengths between fibre and fibre bundle, or the degree that the size effect becomes attenuated or suppressed in the fibre bundle compared with single fibres. Since it is easy to prove that $\Phi > 1$, it reveals there is a greater size effect on the fibre strength than on the bundle strength, or a higher size effect attenuation on the bundle strength.
3. Φ is determined solely by the fibre shape parameter β. Therefore, β is not only a reflection of strength variation along the fibre, but also a measure of the fibre strength translation efficiency, and of the size effect attenuation on fibre bundle strength.
4. It is clear from the above results that one cannot achieve a high strength translation efficiency (a smaller Φ value) and a low size effect (a higher Φ value) at the same time when using fibres to form a fibre bundle, and a compromise has to be reached between the two conflicting requirements.

For better illustration, Fig. 1 shows a graph of Φ against β. The whole curve can be divided into three portions. According to [5], the first portion where $\beta \leqslant 4$ corresponds to the brittle fibres, and the third portion $\beta \geqslant 20$ the ductile fibres. The intermediate portion represents the fibres in between. So a brittle fibre with a high Φ value will have a poor strength translation efficiency, but a good size effect attenuating ability on strength when forming a fibre bundle. In other words, the fibre bundle will have a lower strength but less size effect than the fibre.

3. The size effect in a fibrous structure

It has become known that fibres, once embedded into a composite or twisted into a yarn, will behave differently due to a fibre–matrix or fibre–fibre interaction,

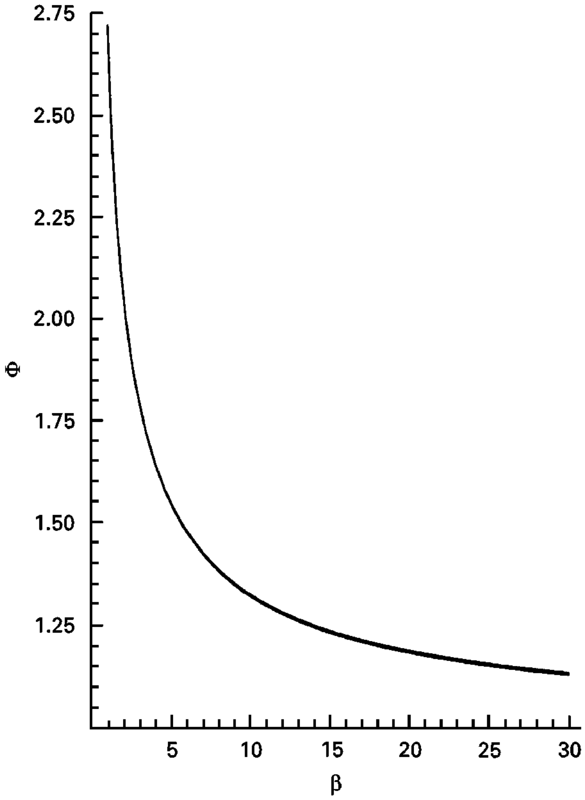

Figure 1 Coleman factor versus fibre Weibull shape parameter β.

and this interaction will inevitably alter the properties of the fibres.

According to Pan [18], if a twisted filament yarn is treated as a chain of twisted fibre bundles of critical length l_c, the expected strength $\overline{\sigma_y}$ for the yarn can be expressed finally as

$$\overline{\sigma_y} = \eta_q V_f (l_c \alpha \beta)^{-1/\beta} \exp\left(-\frac{1}{\beta}\right) \tag{11}$$

where V_f is the fibre-volume fraction in the yarn. η_q is the so-called orientation efficiency factor reflecting the fact that fibres in a twisted yarn are oriented in various directions instead of being parallel to the yarn axis. This factor is a function of q where q is the fibre helix angle at the yarn surface.

The critical length l_c is also given in [18]. If σ_{fb} is the tensile stress which causes the fibre to break, it follows that

$$l_c = \frac{r_f \sigma_{fb}}{\mu g} \tag{12}$$

where r_f is the fibre radius, μ is the frictional coefficient between fibres and g the local lateral pressure.

It is known that for a given breaking strain, the theoretical value of l_c will always be a constant, and so are the values of η_q and V_f. Therefore, it can be deduced from Equation 11 that the yarn strength will be an invariant as well as being independent of the original fibre length l_f.

Furthermore, for a continuous filament yarn, its strength will be enhanced at the initial increase of the twist on the yarn because of the fragmentation process [18]. On the other hand, as the yarn twist level increases, the value of η_q decreases [12] so that the so-called "fibre obliquity" effect will cause the yarn strength to decline at high twist level. Therefore, there is an optimal twist level at which the yarn strength reaches the maximum because of the interactions of the two competing factors.

Finally, it can be readily proved according to statistics theory that the standard deviation of the twisted yarn Θ_y is related to Θ_p in the parallel bundle case as

$$\Theta_y = V_f \eta_q \Theta_p \tag{13}$$

When the yarn surface helix angle $q = 0$, $\eta_q = 1$ and we will have the parallel bundle case. When the fibre-volume fraction $V_f = 1$, the value of Θ_y will reduce to Θ_p.

3.1. Experimental Investigation

A series of tensile tests on single fibres, fibre bundles and yarns were carried out to validate the conclusions drawn from the above theoretical analysis.

3.2. Fibre sample description

Three types of filaments, polypropylene (PP), polyester (PET) and nylon 66 manufactured by BASF were selected for the project. Details of these fibres are provided in Table I.

3.3. Yarn sample preparation and test

All the specimens were prepared and tested according to the standard method ASTM D2101-93 for single fibres, and ASTM D2256-80 for fibre bundles and yarns.

The parallel fibre bundles for each fibre type were taken directly from the filament package. The bundle size (number of filaments in the bundle) was 120 for PP, and 102 for PET and nylon 66. The yarns were prepared using fibre bundles twisted on a twist tester. The twist levels were adjusted according to fibre type as indicated in tables where TF, the twist factor as defined in [13], is used to designate the twist level. When $TF = 0$, there is no twist on the yarn, and the yarn is a parallel fibre bundle. For a given yarn, the higher the TF value, the more twist per length in the yarn.

All specimens were preconditioned prior to test so that the fibres would reach the standard atmospheric

TABLE I Fibre descriptions

	Fibre type		
	PP	PET	Nylon 66
Fibre density ρ_f ($g\,cm^{-3}$)	0.91	1.36	1.14
Fibre denier (g/9000 m)	3.830	5.222	2.144
Fibre Weibull scale parameter α(1/mmGPa$^{\beta}$)	8052.420	2.267×10^7	24.105
Fibre Weibull shape parameter β	13.158	17.509	12.700

equilibrium of 65% RH and 21 °C and the tapes holding specimen ends would be cured adequately to prevent fibre slippage.

All tests were carried out on an Instron testing machine with computerized data acquisition and analysis software. For a gauge length of 50 mm, no less than 30 tests were completed for each result. For other gauge lengths, results were obtained by averaging at least 10 tests. For all different gauge lengths, a constant rate of strain at 100% min was used.

4. Analysis of test results

The strengths of the fibre types tested at four different gauge lengths are shown in Table II, and the results for the yarns made from these fibres at four gauge lengths and four twist levels are given in Tables III, IV and V. As mentioned previously, the yarn strength at the twist level $TF = 0$ gives the fibre bundle strength.

From the data, several observations can be made. First, the fibre strengths are always greater than the corresponding bundle strengths as expected. The same can be said also about their standard deviations as reflected in Equations 3 and 6.

The yarn strength is also always smaller than the fibre strength, and the discrepancies depend on the extent to which the fragmentation process reaches saturation. The yarn strength also can be lower than the bundle strength if the fibre obliquity effect becomes dominant. It can be seen from Tables III, IV and V that there is indeed an optimal twist level, around $TF = 20$, where maximum yarn strength is achieved for all three fibre types.

In addition, the "Coleman factor" values defined in Equation 10 are calculated in Table VI for the three fibre types as well as at different gauge lengths. In the table, the theoretical values Φ_t of the "Coleman factor" which as seen in Equation 10 is supposedly length-independent are included in the parentheses, and the values of the "Coleman factor" calculated using the experimental data corresponding to different gauge lengths are listed in the table. It can be seen that the

TABLE II Fibre strengths and their SD values (GPa)

	Gauge length (mm)			
	10	20	50	100
PP	0.381 (0.054)	0.375 (0.036)	0.361 (0.033)	0.304 (0.035)
PET	0.304 (0.016)	0.302 (0.014)	0.295 (0.020)	0.285 (0.018)
Nylon	0.619 (0.026)		0.549 (0.054)	0.545 (0.033)

TABLE III PP yarn strengths and their SD values (GPa)

	Gauge length (mm)			
	5	10	50	100
$TF = 0$	0.356 (0.020)	0.343 (0.016)	0.320 (0.025)	0.299 (0.010)
$TF = 20$	0.374 (0.023)	0.373 (0.014)	0.360 (0.037)	0.360 (0.050)
$TF = 40$	0.317 (0.018)	0.308 (0.025)	0.302 (0.028)	0.294 (0.023)
$TF = 60$	0.242 (0.035)	0.175 (0.034)	0.240 (0.042)	0.223 (0.058)

TABLE IV PET yarn strengths and their SD values (GPa)

	Gauge length (mm)			
	5	10	50	100
$TF = 0$	0.253 (0.013)	0.248 (0.008)	0.245 (0.008)	0.239 (0.008)
$TF = 10$	0.289 (0.011)	0.275 (0.013)	0.271 (0.010)	0.265 (0.007)
$TF = 20$	0.287 (0.008)	0.296 (0.010)	0.282 (0.009)	0.297 (0.025)
$TF = 40$	0.294 (0.011)	0.295 (0.010)	0.288 (0.009)	0.272 (0.008)

TABLE V Nylon yarn strengths and their SD values (GPa)

	Gauge length (mm)			
	5	10	50	100
$TF = 0$	0.540 (0.012)	0.537 (0.008)	0.490 (0.013)	0.483 (0.007)
$TF = 5$	0.549 (0.010)	0.537 (0.017)	0.492 (0.027)	0.508 (0.009)
$TF = 10$	0.547 (0.007)	0.528 (0.015)	0.520 (0.011)	0.513 (0.007)
$TF = 20$	0.547 (0.007)	0.558 (0.011)	0.501 (0.013)	0.526 (0.022)

empirical "Coleman factor" values remain roughly constant for the different gauge lengths, supporting the theoretical prediction. However, the theoretical Φ_t value is in general greater than the experimental result for every fibre type for reasons still to be explored. In other words, the theory predicts a higher fibre strength translation efficiency and a lower size effect sensitivity, than the actual levels. Because $\Phi > 1$ in Table VI for all cases, it reflects on the one hand that the fibre strength is greater than the bundle strength, and on the other hand that the size effect in a fibre bundle is always less significant than that in a single fibre.

To facilitate the discussion, we have normalized all the strength data, as shown in Tables VII, VIII and IX using each respective value at the common lowest gauge length 10 mm. The data in these tables basically provide indications of strength–gauge length sensitivity for the specimens.

It can be said, based on the data, that the size effect on yarn strength does exist, but the effect is in general smaller in comparison with those either on fibre bundle or on single fibre. Take PP fibre type in Table VII as an example. When gauge length increases 10 times from 10 mm to 100 mm, the fibre strength drops by more than 20%, whereas the fibre bundle strength reduces by 13%, consistent with the theoretical prediction that the size effect on bundle strength is less significant than on fibre strength. The yarn strength on the other hand at $TF = 20$ reduces by only 4%. A similar trend can be seen in the case of PET and nylon fibre types of Tables VIII and IX, respectively, except that irregular fluctuation of yarn strength occurs in these cases so that the yarn strengths occasionally are not the highest at the shortest gauge length. This irregularity may be partly caused by experimental error, but it also provides further evidence that the length or size effect is not as dominant on yarn strength as it is on fibre bundle or fibre strength. Experimental errors occur in all three cases. However, the size effect on fibre bundle or fibre strength is so prevailing that it overshadows the influence of possible experimental errors so that we can see a definite strength dependence on sample length. Another cause for the fluctuation of yarn strength is likely caused by the probabilistic nature of yarn fracture process during which load sharing between the broken and still-surviving fibres, the impact and stress concentration brought in by individual fibre breakage all add uncertainty into the results, leading to the irregularities shown above.

TABLE VI The theoretical and experimental "Coleman factor" values

	Gauge length (mm) Φ		
	10	50	100
PP ($\Phi_t = 1.262$)	1.111	1.128	1.017
PET ($\Phi_t = 1.210$	1.226	1.204	1.192
NYLON ($\Phi_t = 1.269$)	1.153	1.120	1.128

TABLE VII PP fibre and yarn relative strengths

	Gauge length (mm)			
	10	20	50	100
PP fibre	1.000	0.984	0.948	0.798
$TF = 0$	1.000		0.933	0.872
$TF = 20$	1.000		0.965	0.965
$TF = 40$	1.000		0.981	0.955
$TF = 60$	1.000		1.371	1.274

TABLE VIII PET fibre and yarn relative strengths

	Gauge length (mm)			
	10	20	50	100
PET fibre	1.000	0.993	0.970	0.938
$TF = 0$	1.000		0.988	0.964
$TF = 10$	1.000		0.985	0.964
$TF = 20$	1.000		0.953	1.003
$TF = 40$	1.000		0.976	0.922

TABLE IX Nylon fibre and yarn relative yarn strengths

	Gauge length (mm)			
	10	20	50	100
Nylon fibre	1.000		0.887	0.880
$TF = 0$	1.000		0.912	0.899
$TF = 20$	1.000		0.916	0.946
$TF = 40$	1.000		0.985	0.972
$TF = 60$	1.000		0.898	0.943

4.1. Some explanations for the size effect on yarn strength

According to the theory in Equation 11, the yarn strength is determined by the critical length l_c, a constant for a given structure, and should be independent of the original fibre length. This is, however, sometimes in disagreement with the experimental data in the tables. The main reason responsible for this contradiction can be found from the fact that the critical length l_c in a real fibrous structure is in fact not a constant, but follows a certain distribution.

First, in the case where the fibrous structure fails before the fragmentation process reaches saturation, the final lengths of the fibre fragments will be longer than l_c and will not be constant but statistically distributed. This variation of the fibre fragment lengths will lead to the variation of the yarn strength at different yarn cross-sections. Then the strength of the structure is determined by the strength of its weakest cross-section which is, according to the weakest link theory, related to the yarn length. This stochastic nature of the fibre fragment length will also explain the fluctuation of the yarn strength.

Furthermore, during the extension of a fibrous structure, even if the fragmentation process indeed reaches saturation, by definition any fibre fragment with length longer than l_c is still able to break somewhere along its centre section as its stress exceeds its current strength. So the actual fragment lengths vary in the range of $l_c/2$ to l_c [14] [17]. Then for the same reason as above, this variation will cause minor size effect and fluctuation on yarn strength.

To sum up, the size effect for a fibrous structure can be attributed to the size effects brought in by fibres, fibre–fibre interactions, the fragmentation process, and non-uniformity associated with local failure, local property or state perturbations which can be treated as macro-defects distributed along the structure.

5. Conclusions

This study confirms that there is a size effect existing on the strength of a fibrous structure including composite and yarns. It is caused mainly by the variation of the critical length l_c, and is less significant compared with the size effect on either bundle or fibre strength.

It has been demonstrated that the Coleman factor Φ not only reflects the translation efficiency from fibre strength into bundle strength, but also the difference of sensitivity of the size effect on the strengths between fibre and fibre bundle, or the degree that size effect being attenuated or suppressed in bundles compared with that in single fibres. A higher Φ value indicates a greater discrepancy between fibre and bundle strengths, i.e. a lower translation efficiency of fibre strength into bundle strength, but a greater size effect on the fibre strengths than on the bundle strength, or a higher size effect attenuation on the bundle strength.

Because Φ is determined solely by the fibre shape parameter β, this parameter, hence, is not only a reflection of strength variation along fibre, but also a measure of the fibre strength translation efficiency and of the size effect attenuation on bundle strength.

It is clear from the above results that one cannot achieve a high strength translation efficiency (a smaller Φ value) and a low size effect (a higher Φ value) when using fibres to form a fibre bundle, and a compromise has to be reached between the two requirements.

References

1. Z. P. BAZANT, Z. Z. LI and M. THOMA, *J. Engng. Mech. ASCE* **121** (1995) 620.
2. Z. P. BAZANT and R. DESMORAT, *ibid.* **120** (1994) 1945.
3. I. J. BEYERLEIN and S. L. PHOENIX, *Comp. Sci. Technol.* **56** (1996) 75.
4. T. W. CHOU, in "Microstructural design of fibre composites" (Cambridge University Press, Cambridge, 1992) p. 105.
5. *Idem., ibid.* p. 103.
6. W. A. CURTIN, *Polym. Comp.* **15** (1994) 474.
7. *Idem., J. Amer. Ceram. Soc.* **77** (1994) 1072.
8. *Idem., ibid.* **74** (1991) 2837.
9. H. E. DANIELS, *Proc. Roy. Soc.* **A183** (1945) 405.
10. M. R. GURVICH and R. B. PIPES, *Comp. Sci. Technol.* **55** (1995) 93.
11. D. G. HARLOW and S. L. PHOENIX, *J. Mech. Phys. Solids* **39** (1991) 173.
12. J. W. S. HEARLE, P. GROSBERG and S. BACKER, "Structural mechanics of yarns and fabrics", Vol. 1 (Wiley-Interscience, New York, 1969) p. 216.
13. *Idem., ibid.* p. 67.
14. R. B. HENSTENBURG and S. L. PHOENIX, *Polym. Comp.* **10** (1989) 389.
15. C. Y. HUI, S. L. PHOENIX, M. IBNABDELJALIL and R. L. SMITH, *J. Mech. Phys. Solids* **43** (1995) 1551.
16. M. IBNABDELJALIL and S. L. PHOENIX, *Acta Metall. Mater.* **43** (1995) 2975.
17. A. N. NETRAVALI, R. B. HENSTENBURG, S. L. PHOENIX and P. SCHWARTS, *Polym. Comp.* **10** (1989) 226.
18. N. PAN, Prediction of statistical strengths of twisted fibre structures, *J. Mater. Sci.* **28** (1993) 6107.
19. N. PAN, H. C. CHEN, J. THOMPSON, M. K. INGLESBY, S. KHATUA and S. H. ZERONIAN, *J. Mater. Sci.* **32** (1997) 2677.
20. F. T. PEIRCE, *J. Text. Inst.* **17** (1926) 355.
21. H. J. PHILLIPS, R. A. SHENOI and S. M. LEWIS, *Mater. Sci. Lett.* **21** (1994) 229.
22. S. L. PHOENIX and R. RAJ, *Acta Metall. Mater.* **40** (1992) 2813.
23. V. B. SAPOZHNIKOV and M. G. GOLDINER, *J. Phys. A Math. Gen.* **23** (1990) 5309.
24. H. R. SCHWEITERT, and P. S. STEIF, *Int. J. Solid Struct.* **28** (1991) 283.
25. A. S. WATSON and R. L. SMITH, *J. Mater. Sci.* **20** (1985) 3260.
26. W. WEIBULL, *J. Appl. Mech.* **18** (1951) 293.
27. Y. W. ZENG and S. MERIANI, *J. Appl. Crystallogr.* **27** (1994) 782.
28. C. ZWEBEN, *Composites* **25** (1994) 451.

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