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Optimization principles for convective heat transfer

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ABSTRACT

Optimization for convective heat transfer plays a significant role in energy saving and high-efficiency utilizing. We compared two optimization principles for convective heat transfer, the minimum entropy generation principle and the entransy dissipation extremum principle, and analyzed their physical implications and applicability. We derived the optimization equation for each optimization principle. The theoretical analysis indicates that both principles can be used to optimize convective heat-transfer process, subject to different objectives of optimization. The minimum entropy generation principle, originally derived from the heat engine cycle process, optimizes the convective heat-transfer process with minimum usable energy dissipation focusing on the heat–work conversion. The entransy dissipation extremum principle however, originally for pure heat conduction process, optimizes the heat-transfer process with minimum heat-transfer ability dissipation, and therefore is more suitable for optimization of the processes not involving heat–work conversion. To validate the theoretical results, we simulated the convective heat-transfer process in a two-dimensional foursquare cavity with a uniform heat source and different temperature boundaries. Under the same constraints, the results indicate that the minimum entropy production principle leads to the highest heat–work conversion while the entransy dissipation extremum principle yields the maximum convective heat-transfer efficiency.

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1. Introduction

It has been estimated that, of all the worldwide energy utilization, more than 80% involves the heat-transfer process, and the thermal engineering has for a long time recognized the huge potential for conserving energy through heat-transfer efficiency techniques [1–7]. In addition, since the birth of electronic technology, electricity-generated heat in electronic devices has frequently posed as a serious problem [8,9], and effective cooling techniques are hence needed for reliable electronic device operation and an increased device lifespan. In general, approaches for heat-transfer enhancement have been explored and employed over the full scope of energy generation, conversion, consumption and conservation. Design considerations to optimize heat transfer in manufactured systems have often been taken as the key for better energy utilization and have been evolving into a well-developed knowledge branch in both physics and engineering.

During the last several decades and promoted by the worldwide energy shortage, a large number of convective heat-transfer enhancement technologies have been developed including using extended surfaces, spoilers, stirrers, and external electric or magnetic field [1–3,10–12], and they have successfully cut down not only the energy consumption, but also the cost of equipment itself. However, comparing with other scientific issues, engineering heat transfer is still considered to be an experimental problem and most approaches developed are empirical or semi-empirical with no adequate theoretical bases [13]. For instance, for a given set of constraints, it is nearly impossible to design a heat-exchanger rig with optimal heat-transfer performance so as to minimize the energy consumption. Therefore, a much better understanding of the nature of the heat-transfer phenomenon is imperative before more solid methodologies in heat-transfer optimization can be developed.

Nonequilibrium (irreversible) thermodynamics [14], as a phenomenological macroscopic field theory concerned with states and processes in systems out of equilibrium, gives a unified treatment of transport phenomenon, heat transfer included, in continuous media. Historically, nonequilibrium thermodynamics are found in phenomenological laws such as the viscous flow proposed by Newton, heat conduction by Fourier, diffusion by Fick and electrical conduction by Ohm. The general theory of nonequilibrium was elaborated and greatly advanced by Onsager [15,16], in

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Nomenclature

A, B, C_0, A', B', C_0'	Lagrange multiplicator
c_p	specific heat capacity, J kg^{-1} K^{-1}
F	additional volume force per unit volume, N m^{-3}
n	outward normal unit vector
P	pressure, Pa
Q	heat flux across a certain area, W
q	density of heat flow rate, W m^{-2}
$\dot{Q}$	total heat generation rate, W
$\dot{q}$	heat generation rate per unit volume, W m^{-3}
s_g	entropy generation rate per unit volume, W K^{-1} m^{-3}
T	temperature, K
U	velocity vector
u, v, w	velocity component in x, y and z directions, m s^{-1}
V	volume, m^3
x, y, z	Cartesian coordinates, m
ρ	density, kg m^{-3}
μ	dynamic viscosity, kg m^{-1} s^{-1}
λ	thermal conductivity, W m^{-1} K^{-1}
ε	exergy transfer efficiency
ϕ_h	entransy dissipation rate per unit volume, W K m^{-3}
ϕ_m	viscous dissipation rate per unit volume, W m^{-3}
Ω	heat-transfer area
∇	divergence operator
$\prod$	Lagrange function

Subscript

0	ambient
b	boundary
L	left boundary
R	right boundary
in	inlet
out	outlet

terms of introducing certain system relations and deriving the principle of the least energy dissipation. Overall, nonequilibrium thermodynamics is developed with the establishment of a set of general balance equations, of which the entropy balance equation plays a central role in the theory of irreversible processes, for entropy generation is held as a measure of the irreversibility for such processes.

Furthermore Prigogine [17] developed the principle of minimum entropy production based on the idea that the entropy generation of a thermal system at steady state had to be at the minimum [18,19]. In 1970s, a branch of modern thermodynamics, the finite-time thermodynamics, was developed by Berry, Andersen and Salamon [20–22], which investigates the performance limits and the optimization of thermodynamic processes with finite operating time. Other concepts, such as the maximum power, generalized potential, and thermodynamic length, have been successfully applied to practical optimization of thermodynamic cycles. In parallel, Bejan [23,24] introduced the entropy generation function for estimating the irreversibility of convective heat-transfer processes in a system along finite temperature gradient with viscous effects. Then convective heat-transfer processes were optimized with the objective of minimum entropy generation. Based on this method, several researchers, including Nag and Mukherjee [25], Sahin [26], Demirel [27,28], Sara [29] and Ko [30], analyzed the influences of geometrical, thermal and flow boundary conditions on the entropy generation in various convective heat-transfer processes, and then optimized them based on the premise that the minimum entropy generation will lead to the most efficient heat-transfer performance, despite the fact that this premise has been called into question by some researchers [31–33].

As we know, when heat transferred from a higher temperature to a lower one, although the total quantity of the thermal energy is conserved, this same amount of energy is unable to cause a reverse of this heat flow without additional external energy. In other words, the *heat-transfer ability* of the same energy is reduced by the "inherent equivalent thermal resistance" after the process. Although the concept of entropy was originally proposed just to describe such an "energy losing the capacity to do work" scenario, this entropy concept is more appropriate to a *heat energy conversion to work* process, not to a *heat-transfer* process. It is for this very reason that the minimum entropy principle is considered incapable to be used as a criteria for optimization of a heat-transfer process [31–33]. Consequently a new physical quantity, termed as entransy introduced by Guo et al. [34], is regarded as a likely solution for the problem. In this new theory, the concept of *entransy dissipation* was defined, in place of entropy production, and used to measure the irreversibility of a heat-transfer process. Furthermore, Guo et al. [34] proposed the entransy dissipation extremum principle for heat-transfer optimization: the extremum entransy dissipation corresponds to the minimum transfer resistant for a given heat-transfer problem. In reference [35], the concept of entransy was originally referred to as the heat-transfer potential capacity, and the entransy dissipation extremum principle was considered equivalent to the dissipation extremum principle of the potential capacity. Based on this theory, several types of heat-transfer processes, including heat conduction [34], convective heat transfer [36–39] and thermal radiation [40], have been analyzed and optimized numerically, and a few heat-transfer enhancement approaches have been developed, e.g. utilization of the alternating elliptical axis tubes [39] and the discrete doubled inclined rib tubes [38].

From the aforementioned discussion, there exist two potential methods for convective heat-transfer optimization: one is the minimum entropy generation (MEG) principle developed by Bejan [23,24], viewed as the *thermodynamic* optimization [24–26], and the other is the entransy dissipation extremum (EDE) principle introduced by Guo et al. [34], referred to here as the *heat-transfer* optimization. Thus, the objective of this contribution is to investigate the physical essentials and the applicability of, as well as the differences between, these two principles in convective heat-transfer optimization. First, different Euler's equations, i.e. the optimization equations, are derived corresponding to these two principles, then a convective heat transfer process in a rectangular cavity with uniform internal heat source is taken as a testing case to be optimized using the two Euler's equations, respectively, and finally the *two optimized* results are compared side by side. In addition, a new concept of *available energy transfer efficiency* is introduced to further elucidate and illuminate the two principles.

2. Convective heat-transfer optimization methods

In order to compare the MEG and EDE principles, we derive the optimization equation for each in this section using the basic governing equations of convective heat-transfer process.

2.1. The MEG based method

According to the MEG principle [23–27,29,30], there exist two irreversible processes during convective heat transfer: the momentum transport and the heat transfer, and both contribute to the entropy generation. Therein, the local entropy generation function induced by the heat transfer over finite temperature difference is

$$s_g = (\lambda \nabla T \cdot \nabla T)/T^2. \tag{1}$$

For a convective heat-transfer process with a given rate of viscous dissipation, the optimization objective here is to find an optimal velocity and temperature distribution, which lead to the minimum entropy generation. By the calculus of variations, a Lagrange function is first constructed with given constraints, including the mass conservation, energy conservation and fixed viscous dissipation rate

$$\prod{}' = \iiint_{\Omega} \Big[(\lambda \nabla T \cdot \nabla T)/T^2 + C_0'\phi_m + A'(\nabla \cdot \lambda \nabla T + \dot{q} - \rho c_p U \cdot \nabla T) + B' \nabla \cdot \rho U \Big] dV \tag{2}$$

where A', B' and C_0' are Lagrange multipliers. A' and B' vary with position, while C_0' is constant for a given viscous dissipation. ϕ_m is the viscous dissipation function

$$\phi_m = \mu \left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2 \right] \tag{3}$$

The variation of Eq. (2) with respect to velocity U yields

$$\mu \nabla^2 U + \frac{\rho c_p}{2C_0'} A' \nabla T + \frac{1}{2C_0'} \nabla B' = 0. \tag{4}$$

The variation of Eq. (2) with respect to temperature T is

$$-\rho c_p U \cdot \nabla A' = \lambda \nabla \cdot \nabla A - 2\frac{\lambda}{T} \nabla \cdot \left(\frac{\nabla T}{T}\right), \tag{5}$$

and the boundary conditions for the variable A' include $A_b' = 0$ for given temperatures at boundaries, and $(\partial A'/\partial n)_b = 2(\partial T/\partial n)_b/(T^2)_b$ for given heat flow rates.

There are four unknown variables and four governing equations including Eqs. (4) and (5), the continuity equation and the energy equation, so the unknown variables can be solved for prescribed boundary conditions. Meanwhile, the fluid flow must also meet the momentum equation

$$\rho U \cdot \nabla U = -\nabla P + \mu \nabla^2 U + F. \tag{6}$$

Comparing Eqs. (4) and (5) gives the following relations

$$B' = -2C_0' P, \tag{7}$$

$$F = C_\phi' A' \nabla T + \rho U \cdot \nabla U, \tag{8}$$

where C_ϕ' is associated with the viscous dissipation

$$C_\phi' = \frac{\rho c_p}{2C_0'}. \tag{9}$$

Substituting Eqs. (8) and (9) into 6 gives

$$\rho U \cdot \nabla U = -\nabla P + \mu \nabla^2 U + (C_\phi' A' \nabla T + \rho U \cdot \nabla U), \tag{10}$$

This is the Euler's equation, essentially the momentum equation with a special additional volume force defined in Eq. (9), by which the fluid velocity pattern is adjusted to lead to an optimal temperature field with minimum entropy generation during a convective heat-transfer process. For a given C_ϕ, solving Eqs. (5) and (10), the combined continuity equation and the energy equation results in the optimal flow field with the minimum entropy generation for given viscous dissipation and boundary conditions.

2.2. The EDE based method

For convective heat transfer in media with an inner heat source, the thermal energy conservation equation is

$$\rho c_p U \cdot \nabla T = \nabla \cdot (\nabla T) + \dot{q} \tag{11}$$

Multiplying both sides of Eq. (11) by the temperature, T, yields

$$U \cdot \nabla G = \nabla \cdot (\lambda T \nabla T) + \dot{q} T - \lambda |\nabla T|^2 \tag{12}$$

where $G = \rho c_p T/2$, defined as *entransy* by Guo et al. [34] because it represents the heat-transfer ability of a thermal capacitor. The entransy has the meaning of "energy" of heat similar to the electrical energy for an electrical capacitor. Based on this definition, the left hand of Eq. (12) means the entransy flow, while the first term on the right is the entransy diffusion caused by thermal diffusion, the second is the entransy input by the inner heat source, and the last one can therefore be defined as the dissipation function of entransy, which represents the heat-transfer ability lost during the heat-transfer process.

In convective transfer processes, though the heat energy is conserved, as indicated by Eq. (11), the entransy of the system reduces, as shown in Eq. (12). The local entransy dissipation function, which is another scale to measure the irreversibility of the heat-transfer process, is therefore defined as [34]

$$\phi_h = -q \cdot \nabla T = \lambda \nabla T \cdot \nabla T. \tag{13}$$

It is to be noted that in a heat-transfer process, when boundary temperature differences are given, optimizing the heat transfer is equivalent to maximizing the heat flux q and thus to *maximizing* the entransy dissipation; whereas when the boundary heat flow rates are given, heat-transfer optimization is to minimize the temperature gradient ∇T, which in turn will lead to *minimizing* the entransy dissipation.

Thus, for given viscous dissipation, the optimization objective here is to find out optimal velocity and temperature fields which will result in the *extremums* of entransy dissipation during a convective heat-transfer process. Meanwhile, the heat flow is also governed by the continuity equation and the energy equation. Similar to the derivation in the MEG principle in last section, the constraints can be removed by using the Lagrange multipliers to construct a function

$$\prod = \iiint_{\Omega} \big[\lambda \nabla T \cdot \nabla T + C_0 \phi_m + A(\nabla \cdot \lambda \nabla T + \dot{q} - \rho c_p U \cdot \nabla T) + B \nabla \cdot \rho U \big] dV, \tag{14}$$

where A, B and C_0 are the Lagrange multipliers. Because of the different types of the constraints, A and B vary with position, while C_0 remains constant for a given viscous dissipation.

Likewise, the variational of Eq. (14) with respect to A and B yields the energy equation and the continuity equation, respectively. Yet the variational of Eq. (14) with respect to velocity U offers

$$\mu \nabla^2 U + \frac{\rho c_p}{2C_0} A \nabla T + \frac{1}{2C_0} \nabla B = 0. \tag{15}$$

The variational of Eq. (14) with respect to temperature T is

$$-\rho c_p U \cdot \nabla A = \nabla \cdot \lambda \nabla A - 2 \nabla \cdot \lambda \nabla T, \tag{16}$$

and the boundary conditions of the variable A are $A_b = 0$ for given boundary temperatures, and $(\partial A/\partial n)_b = 2(\partial T/\partial n)_b$ for given boundary heat flow rates.

Comparing the corresponding momentum equations, i.e. Eq. (6), and Eq. (15), gives the following relations for the same system

$$B = -2C_0 P, \tag{17}$$

$$F = C_\Phi A \nabla T + \rho U \cdot \nabla U, \tag{18}$$

where C_ϕ is related to the viscous dissipation as

$$C_\Phi = \frac{\rho c_p}{2C_0}. \tag{19}$$

Substituting Eqs. (17) and (18) into Eq. (5) gives

$$\rho U \cdot \nabla U = -\nabla P + \mu \nabla^2 U + (C_\Phi A \nabla T + \rho U \cdot \nabla U). \tag{20}$$

This is the Euler's equation governing the fluid velocity and temperature fields under the EDE principle during a convective heat-transfer process. Eq. (20) is similar to its counterpart Eq. (10), in that both are the momentum equations with an additional volume force. However as shown in Eqs. (5) and (16), the governing equation of the variable A associated with EDE is different from that of the variable A' with the MEG. This difference will result in two dissimilar additional volume forces in each Euler's equation, and thus lead to two different velocity and temperature fields at the end. By comparing these two different optimization results, both the applicability and the physical essentials of the MEG principle and the EDE principle can be analyzed.

In summary, the concepts of entropy and entropy generation were originated from the discipline of thermodynamics with the conversion efficiency between the heat and the work as the performance criteria of a cycle. The entropy thus reflects the *heat–work conversion ability* of a system, and the entropy generation describes the dissipation of such conversion ability during thermodynamic processes, so the entropy generation represents the irreversibility of heat-transfer processes only in terms of the energy dissipation. That is why heat-transfer optimization based on the minimum entropy generation principle is usually called the thermodynamic optimization [24–26].

There are however a number of heat-transfer processes which do not involve thermodynamic cycles, such as cooling of electronic equipment or heat convection of radiators. In these cases, heat-transfer design is usually concerned with the rate and/or efficiency at which heat energy is transferred across the system boundaries. In order to achieve this goal, we need new physical quantities to describe the *heat-transfer ability* and its dissipation of a system during heat transfer. The concepts of entransy and entransy dissipation have shown to be the very quantities for such purpose.

In addition, from the expression of the entropy generation function in Eq. (1), the entropy generation rate is determined not only by the temperature gradient but also the absolute temperature, which clearly contradicts with the phenomenological law of heat transfer, i.e. the Fourier's law, whereas the entransy dissipation rate is simply a function of the temperature gradient for a given thermal conductivity.

3. Numerical results and discussion

In engineering, many physical or chemical processes are accompanied by heat generation or absorption, and mechanical stirring in fluids is often employed to enhance the heat-transfer rate and efficiency. Stirring is a typical example where convection is introduced to boost heat transfer by constantly change the temperature profile and fluid velocity field in the stirrer. Delaplace [10], Karcz [11], Schafer [12], Cudak [41], Cho [42] and Yapici [43] analyzed the influences of configurations, rotating velocity, frequency, eccentrically located impeller on the heat-transfer coefficient, and then optimize the above parameters to obtain a better fluid flow pattern with a relative higher heat-transfer coefficient. As we know, the flow pattern determines the convective heat-transfer performance for a given fluid and certain boundary conditions, however in most cases the best flow pattern is unknown before optimization. Therefore, most optimizations and designs have to be empirical and expensive. Herein, the aforementioned two optimization principles will be used in this section to first demonstrate their capacity in treating this engineering problem so as to shed some predictive light on stirrer designing, and meanwhile to illustrate the differences between them.

For simplicity, consider a two-dimensional convective heat-transfer process in a foursquare cavity with the length of side $L = 100$ mm as shown in Fig. 1. The boundary has two sections at

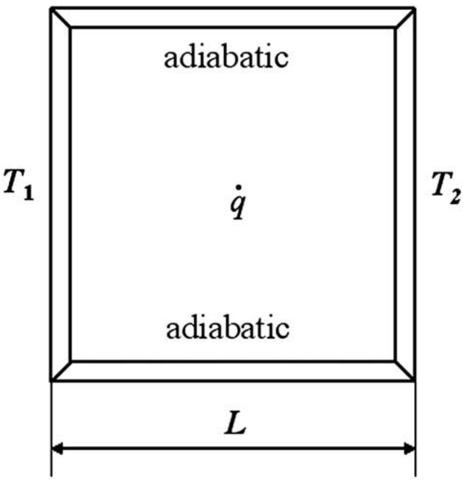

Fig. 1. Sketch of the foursquare cavity geometry and boundary conditions.

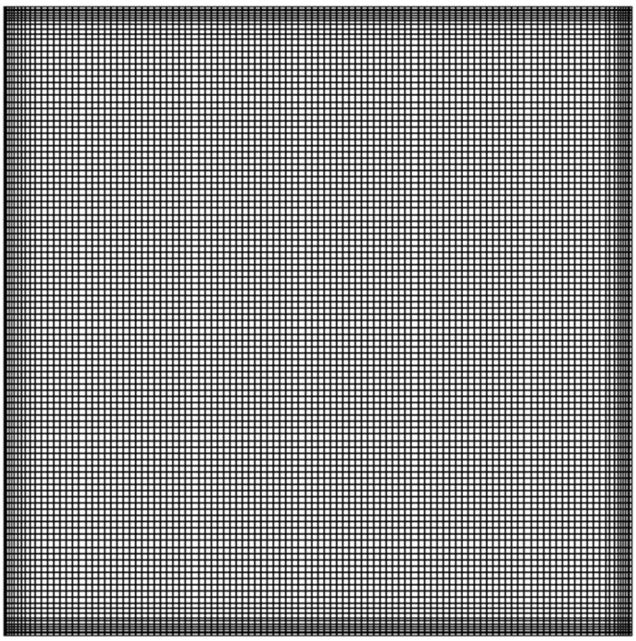

Fig. 2. Computation meshes of the cavity.

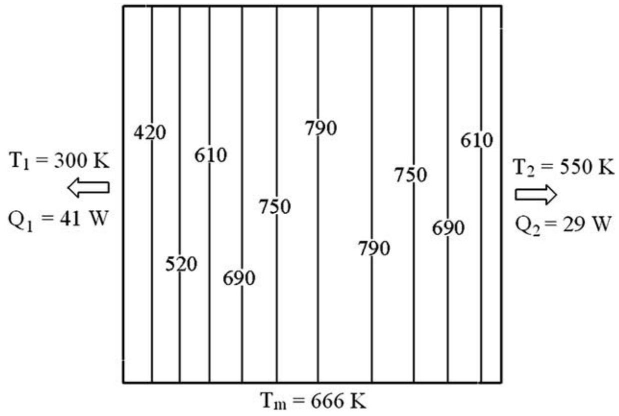

Fig. 3. Temperature contours without any volume force.

temperature $T_1 = 300$ K and $T_2 = 550$ K, respectively, while the other sides are adiabatic. Assume then there exists a uniform heat source with the overall heat generation rate of 70 W in the domain, and the heat generated is transferred to the boundaries through the airflow as the convective fluid. The physical properties of the air include: $\rho = 1.225$ kg m^{-3}, $\mu = 1.7894 \times 10^{-5}$ kg m^{-1} s^{-1}, $\lambda = 0.0242$ W m^{-1} K^{-1}, $c_p = 1006.43$ J kg^{-1} K^{-1}, all remain constant during the process. The optimization objective is to find out a suitable air velocity distribution to minimize the average temperature in the cavity at given viscous dissipation rate.

The CFD program, FLUENT 6.0, was used to solve the conservation equations at the given boundary conditions to obtain the fluid velocity and temperature fields in the cavity. The velocity and pressure were linked using the SIMPLEC algorithm with the convection and diffusion terms discretized using the QUICK scheme. The user defined function (UDF) in FLUENT is utilized for solving the governing equations of the parameters, A and A' with additional volume forces in Eqs. (10) and (20), respectively. Fig. 2 shows the grid distribution. The mesh is more condensed in the near wall regions, where steeper velocity gradient is expected. The total element of the grid is about 20,000.

Without any volume force, i.e. as in Eqs. (8) and (18), the air remains still in the foursquare cavity, and the heat generated in the domain is *conducted* to both left and right boundaries. The temperature contours in the foursquare cavity are shown in Fig. 3, indicating that the averaged temperature in the domain is 666 K, and the heat-transfer rates at the left and the right boundaries are 41 W and 29 W, respectively.

Based on the EDE principle where $C_\phi = -4 \times 10^{-9}$ is calculated from Eq. (20), Fig. 4(a) and (b) are resulted to show the optimized velocity fields and temperature contours in the system. There exist two vortexes closer to the left boundary in Fig. 4(a) due obviously to the convection from the additional volume forces, indicating that more heat will be transferred out of the cavity through the left boundary. As shown in Fig. 4(b), the heat-transfer rate at the left boundary is 54 W, 13 W larger than the result for the pure heat conduction case in Fig. 3. It is also for this reason that the averaged temperature in the domain is cut down to 562 K, i.e. a reduction of 104 K. For prescribed heat-transfer rate, the entransy dissipation definition as shown in Eq. (13) suggests that lower temperature gradient will result in a smaller entransy dissipation rate; the vortexes near the left lower temperature boundary caused by the additional volume force suppress the overall temperature variations in the entire cavity. By integrating the local time rate in Eqs. (1), (3) and (13) over the entire domain with the date processing function in FLUENT, the total time rates of entropy generation and viscous dissipation are 8.2×10^{-2} W K^{-1} and 2.4×10^{-8} W, respectively, while the total entransy dissipation rate is 1.4×10^{4} W K^{-1}, as listed in Table 1.

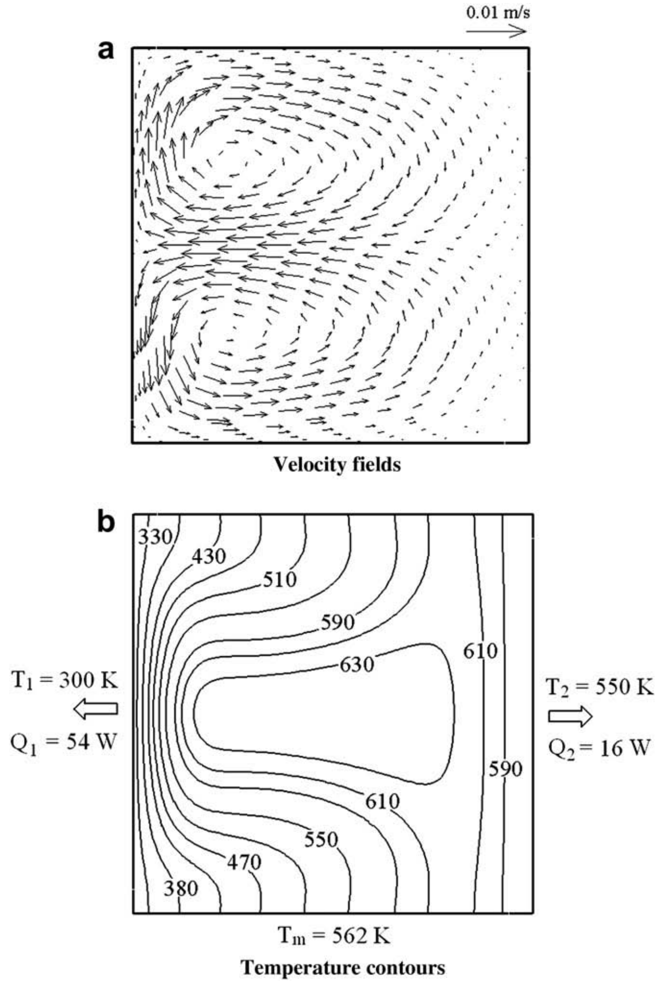

Fig. 4. Optimized velocity and temperature fields based on the entransy dissipation extremum principle ($C_\phi = -4 \times 10^{-9}$).

On the other hand, for the MEG method where $C_\phi' = 0.135$ from Eq. (18), the optimized velocity and temperature fields are shown in Fig. 5(a) and (b), respectively. There are also two vortexes in the foursquare cavity, but their positions are instead more inclined to the right boundary of higher temperature, meaning more heat will

Table 1
The total time rate of viscous dissipation, entropy generation and entransy dissipation optimized based on the MEG and EDE principles.

Optimization principle	$C_\phi(C_\phi')$	viscous dissipation rate ϕ_m (W)	Entropy generation rate S_{gen} (W K^{-1})	Entransy dissipation rate ϕ_h (W K)
EDE	-4×10^{-9}	2.4×10^{-8}	8.2×10^{-2}	1.4×10^{4}
MEG	0.135	2.4×10^{-8}	7.3×10^{-2}	1.7×10^{4}

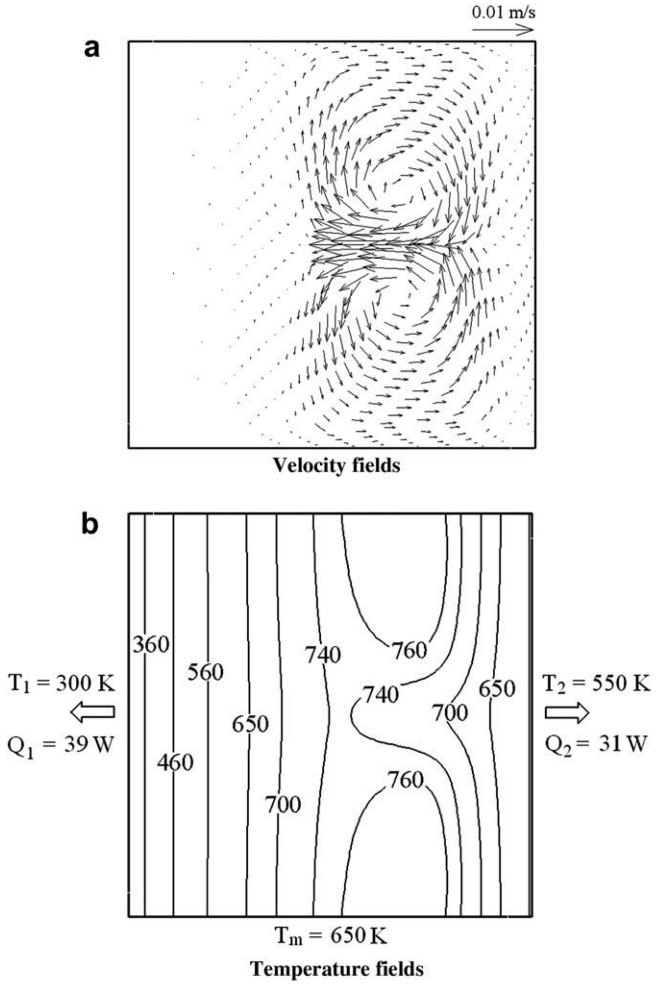

Fig. 5. Optimized velocity and temperature fields based on the minimum entropy generation principle ($C_{\phi}' = 0.135$).

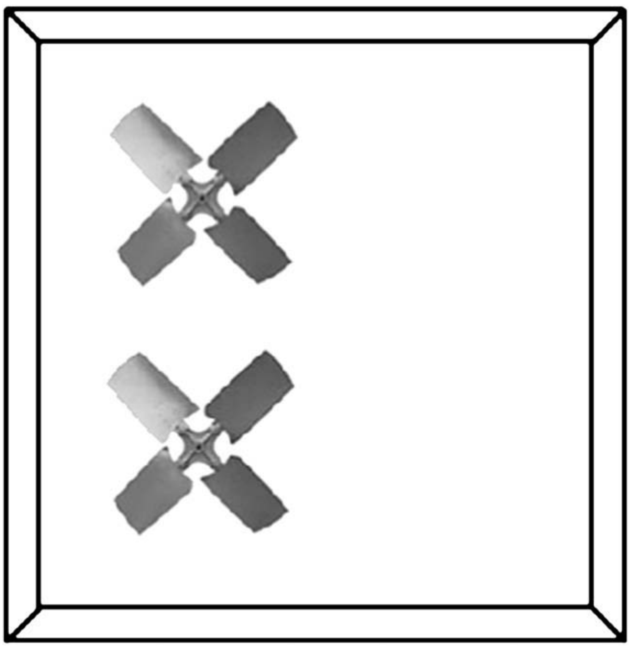

Fig. 6. Sketch of the positions of the stirrers in the foursquare cavity.

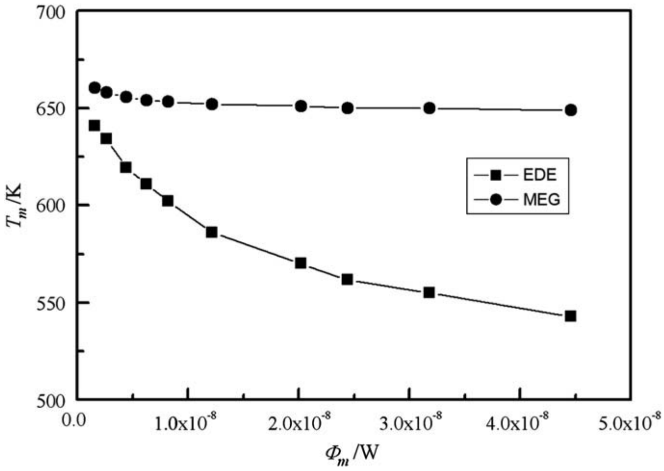

Fig. 7. The averaged temperatures versus value of ϕ_m ($T_2 = 550$ K, $\dot{Q} = 70$ W).

be moved to the right boundary. As shown in Fig. 5(b), the heat transfer at the right boundary is 31 W, 2 W greater than the result for the pure heat conduction condition, and 15 W higher than that optimized using the entransy dissipation extremum principle.

For given heat-transfer rate and temperature gradient, the entropy generation, as defined in Eq. (10), suggests that higher absolute temperature during heat transfer processes will lead to a smaller entropy generation rate, so the entropy generation induced by heat transfer at the right boundary with a higher temperature will be relative small now. Thus, when utilizing the MEG principle, in order to decrease the overall entropy generation in the entire cavity, the additional volume force will be needed to generate the vortexes near the right boundary, thus more heat will be transferred to the right boundary, leading to an average temperature in the cavity of 650 K, 98 K higher than the result by the EDE principle.

In addition, integrating the local time rate over the cavity, the total time rates of viscous dissipation, entropy generation, and entransy dissipation for this case are 2.4×10^{-8} W, 7.3×10^{-2} W K^{-1} and 1.7×10^{4} W K, respectively, as listed in Table 1. By comparison with the optimized results using the EDE principle, we find that for the same viscous dissipation, the total *entropy* generation rate based on the MEG principle is lower than that based on the EDE principle, while the total *entransy* dissipation rate based on the MEG principle is larger than that from the EDE principle. Therefore, the two principles are not equivalent in physics for convective heat-transfer

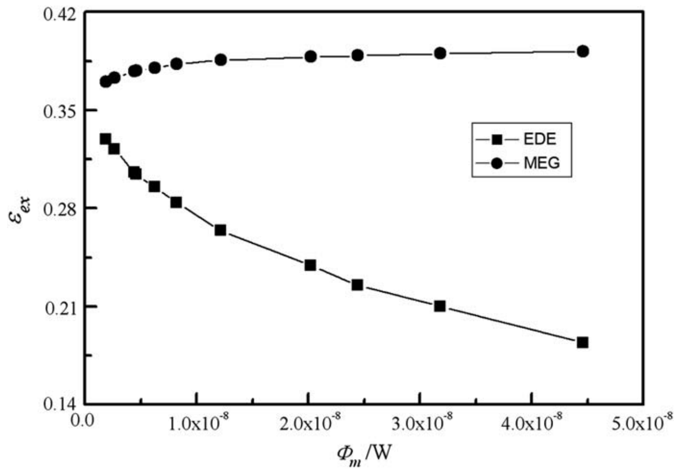

Fig. 8. The exergy transfer efficiencies versus value of ϕ_m ($T_2 = 550$ K, $\dot{Q} = 70$ W).

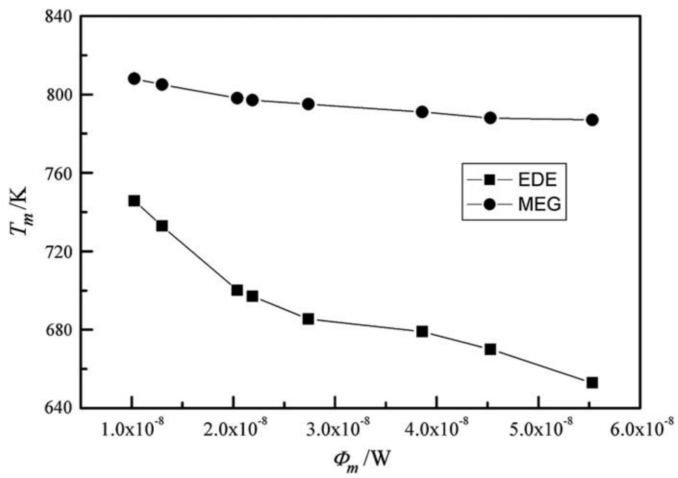

Fig. 9. The averaged temperatures versus value of ϕ_m ($T_2 = 700$ K, $\dot{Q} = 100$ W).

optimization, yielding some differences between the optimized results.

For practical engineering applications, the numerical analysis shows that generating two vortexes near the left boundary can increase the local heat-transfer coefficient near the left boundary, i.e. decrease the temperature difference between the fluid and the cold boundary, and consequently significantly decrease the averaged temperature in the foursquare cavity. Therefore, a mechanical stirrer can be placed near the left boundary, as shown in Fig. 6, to obtain the desired velocity field in Fig. 4(a) to effectively enhance the heat-transfer performance. This design agree qualitatively with the experimental data [41], where the impeller is eccentrically located and the local heat-transfer coefficient at the boundary near the impeller is larger than the other boundaries.

Furthermore, in order to find out the physical essentials lying in the MEG and the EDE principles, the predicated averaged temperature over the entire cavity at difference viscous dissipation rates, ϕ_m, are shown in Fig. 7, indicating that the averaged temperatures obtained by both principles decrease with the viscous dissipation rate, but the dropping speed based on the EDE criterion is substantially greater than that of the MEG. Because the heat generation rate in the cavity remains constant, a steeper drop in average temperature corresponds to a more effective cooling efficiency. That is to say, utilizing the EDE route will optimize convective heat-transfer process to achieve the objective of the maximum heat-transfer coefficient.

Since entransy is a physical quantity describing the heat-transfer ability of a system, while entropy is always relevant to exergy [44], a physical quantity for evaluating the system heat–work conversion ability, we believe it is desirable to define a concept of exergy transfer efficiency,

$$\varepsilon_{ex} = \frac{Ex_{out}}{Ex_{in}}. \tag{21}$$

where, $Ex_{in} = \iiint_{\Omega} \dot{q}(1 - T_0/T)dV$, represents the total exergy inlet, and $Ex_{out} = Q_L(1 - T_0/T_L) + Q_R(1 - T_0/T_R)$ is the exergy outlet.

Fig. 8 shows the predicated exergy transfer efficiencies of the system at different viscous dissipation rates. Along with the increase of the viscous dissipation rate, the exergy transfer efficiency optimized by the MEG principle increases, while that by the EDE principle decreases. Again entropy and entropy generation concepts originated from the discipline of thermodynamics with the conversion efficiency between heat and work, i.e. the utilization efficiency of exergy, as the performance criteria of a cycle. Conversely, entransy and entransy dissipation use the heat-transfer effectiveness as the performance criterion for evaluating the transport efficiency of heat. It is conceivable that utilizing the MEG principle can obtain the result with the maximum exergy transfer efficiency, i.e. the maximum heat–work conversion, but the heat-transfer efficiency is not the maximum.

In order to further illustrate the above conclusions, Figs. 9 and 10 give the predicated averaged temperature and the exergy transfer efficiencies of the system at difference viscous dissipation rates, respectively, with the right boundary temperature of 700 K and the overall heat generation rate of 100 W. Similar to the results shown in Figs. 7 and 8, with increasing the overall viscous dissipation rate, the dropping speed of the averaged temperature based on the EDE criterion is greater than that of the MEG, while the exergy transfer efficiencies optimized by the EDE principle becomes much worse than that of the MEG. Thus, utilizing the EDE principle can obtain the result with the best heat-transfer performance, while utilizing the MEG principle will obtain the maximum the heat–work conversion efficiency.

4. Conclusions

By analyzing the irreversibility of heat-transfer processes and using the calculus of variations, two different Euler's equations, essentially the momentum equations with two different special additional volume forces, have been deduced with the objectives of the minimum entropy generation and the entransy dissipation extremum, respectively, as two different optimization schemes to treat the problem of system heat-transfer efficiency at given constraints. Next a two-dimensional convective heat-transfer process in a foursquare cavity with a uniform heat source and fixed boundary temperatures was taken as a testing case. The velocity and temperature in the cavity were optimized based on the minimum entropy generation principle and the entransy dissipation extremum principle, respectively, from which the averaged temperature, the total entropy generation rate and the total entransy dissipation rate in the cavity during convective heat-transfer processes were calculated, and the following conclusions are drawn.

For given viscous dissipation, utilizing the entransy dissipation extremum principle can obtain the optimal velocity field with two vortexes near the lower temperature boundary, which in turn transfers more heat to the location and noticeably decreases the

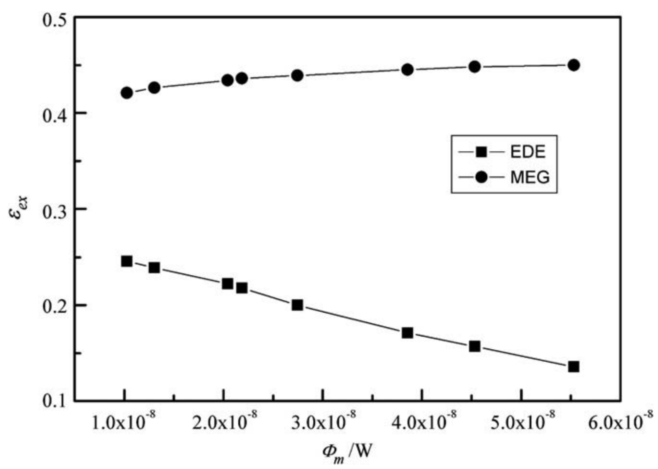

Fig. 10. The exergy transfer efficiencies versus value of ϕ_m ($T_2 = 700$ K, $\dot{Q} = 100$ W).

averaged temperature in the domain, i.e. effectively improve the convective heat-transfer performance. Using the minimum entropy generation principle on the other hand can obtain the velocity field with two vortexes more inclined to the higher temperature boundary, meaning that more heat will be transferred to the higher temperature boundary, resulting in a maximum exergy transfer or maximum heat–work conversion but not the maximum heat-transfer efficiency. In practical engineering applications, the resulted optimal velocity fields can shed some predictive light on stirrer designing for heat exchange operations.

In conclusion, the entransy dissipation extremum principle is more suitable to maximize the heat-transfer performance, while the minimum entropy generation principle is more suitable for maximizing the heat–work conversion.

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