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# Explanation of the Fractal Characteristics of Goose Down Configurations

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**Abstract** Goose down has superior thermal insulating properties as a filler material used widely as an insulator in textile products. Its particular "tree" structure is expected to attribute greatly to this insulating property. In this paper, fractal structures of the down "tree" are observed using scanning electron microscopy (SEM), and the configuration characteristics of goose down are described quantitatively by local fractal dimensions. From two expects, the local fractal dimensions were calculated both theoretically and experimentally, revealing its value to be very close to the golden mean, 1.618. This near-optimal fractal dimension may be attributable to the excellent thermal insulation of goose down assembly, and the potential applications of such a fractal structure are also discussed.

**Key words** goose down, configuration characteristics, fractal geometry, local fractal dimension

Down is a layer of undercoat growing on the cuticular of fowl, such as geese, ducks and swans, etc. Goose down and duck down are thereinto the most usual down, which and very soft and fine in nature. For a long time, down has been used as the best filler material for bedding and outerwear, able to withstand cold climates because of its superior thermal insulating properties. The investigation of the configuration characteristics and thermal insulation mechanisms of down and down assemblies have never stopped, and there are several publications [1–4] on the morphology and structure, as well as physico-chemical properties, of down. However, using mathematical methods and fractal theory to characterize down structure and further to explain heat transfer problems in single down and down assemblies has never been attempted up to now.

In this paper, we attempt to describe the configuration characteristics of goose down quantitatively using tools from fractal geometry and local fractal dimensions, as well as pioneering research on the heat transfer problems and thermal insulation properties of down and down assemblies. Before presenting the structure analysis, a general background of fractal geometry and the essential concepts of the "local fractal dimension" are briefly reviewed in the following section.

## Fractal Geometry and Local Fractal Dimension

Fractal geometry is a new subject that has developed quickly over recent years, which reveals the unifications between in-order and out-of-order, and determinability and randomicity. Fractal structures have aroused a great deal of interest in various fields [5–8].

First, there are great differences between conventional Euclidean geometry and fractal geometry. Euclidean geometry describes regular objects such as points, curves, sur-

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faces and cubes using integer dimensions 0, 1, 2 and 3, respectively. However, most of the objects in nature, such as the surfaces of mountains, coastlines, the microstructure of metals, etc., are not Euclidean objects. Such objects are called fractals, and are described by a non-integral dimension called the *fractal dimension* [9].

For example, if one were to measure the length of a coastline, the length would depend on the size of the measuring stick used. Decreasing the length of the measuring stick leads to a better resolution of the convolutions of the coastline, and as the length of the stick approaches zero, the coastline's length tends to infinity. This is the fractal nature of the coastline. Since a coastline is so convoluted, it tends to fill space, and its fractal dimension lies somewhere between that of a line (which has a value 1) and a plane (which has a value 2).

The measure of a fractal object $N(L)$ (namely length, area or volume) is governed by a *scaling* relationship of the form

$$N(L) \sim L^{d_f} \qquad (1)$$

where the "~" should be read as "scales as" and $d_f$ is the fractal dimension of the object. As an illustration of the above relationship, consider the microstructure of a metal showing grains of various shapes and sizes. This is a fractal object in a two-dimensional plane. It is observed that the average area, $N(L)$, of the grains within squares of different sizes $L \times L$ (defined as the arithmetic mean of only those samples for which the center of the square falls on the grain) *scales* with the length, $L$, over a range of lengths, as per the above relationship [10]. The fractal dimension, $d_f$, calculated as the slope of a log–log plot of $N(L)$ against $L$, lies in the range $1 < d_f < 2$. Alternatively, the microstructure can be described in terms of two *linear* fractal dimensions, each having a value between 0 and 1, along mutually perpendicular directions. The *linear* fractal dimensions are obtained from a scaling relationship as in equation (1), where $N(L)$ denotes the average total length of the intercepts between a measuring line of size $L$, and the microstructure.

Associated with equation (1) is the property of self-similarity, which implies that the $d_f$ calculated from the relationship in equation (1) remains constant over a range of length scales, $L$. Exact fractals such as the Sierpinsky gasket, Koch curve, etc., exhibit self-similarity over an infinite range of length scales [9]. In actual applications, self-similarity in a global sense is seldom observed, and the "fractal" description is usually based on *a statistical self-similarity*, which the objects exhibit in some average sense, over a certain *local* range of the length scale $L$, which is of relevance to the problem [11–13]. The fractal dimension calculated based on the *local, statistical self-similarity* is termed the *local fractal dimension* to distinguish it from the term fractal dimension, which implies global self-similarity at all length scales. The fractal dimensions (local or global) of statistical fractals are usually estimated in the same manner as in the illustration of the microstructure, using a scaling relationship between $N(L)$ and $L$ of the form in equation (1) (see [11]).

The concept of local, statistical self-similarity has been used in many applications ranging from the characterization of the microstructure of materials [10] to the analysis of speech waveforms and signals [13, 14]. For example, in the area of speech recognition in a speech reside in the region of short time scales. While the term local fractal dimension is used by some investigators [14], others refer to it simply as the fractal dimension [10]. We adopt the term local fractal dimension for the analysis of the configuration characteristics of goose down fiber in this paper.

## Local Fractal Description of Goose Down

Figure 1(a) shows the common soft and fine characteristics of down. However, when magnified using scanning electron microscopy (SEM), down is revealed to be a large tree shape. The whole down consists of a short central nucleus from which a number of branches diverge at a wide range of orientations (Figure 1(b)). Compared with the down branch, the down nucleus is very small and could be considered as a dot in the macroscopical picture. On some main branches, analogous secondary branches will shoot out, and keep the same structures as the main branches (Figure 1(c)). In turn, each branch also carries a large number of fibrils, all of which diverge from the branches at about 30° to 90° (Figure 1(d)). The down branches and down fibrils constitute the main body of down. Finally many little triangle nodes and crotches diverge from the fibrils to hold in place a single crossing fibril (Figure 1(e)).

Thus, we can see that the SEM images of goose down exhibit an evident "self-similarity" between the parts of goose down fibers and the whole, and this kind of "self-similarity" is not in a global sense, but based on a *statistical* self-similarity only in some average sense, over a certain local range of the length scale $L$. Magnifying or reducing the figures, the configuration characteristics remain statistically fixed. Thus, it is proved that the goose down has an evident local "fractal tree" structure.

## Evaluation of $d_f$ of Goose Down

### Computer Simulation Prediction of $d_f$

In simulating the morphologic structure and structural parameters of the goose down [15], computer-generated fractal graphics were created using the Basal generating

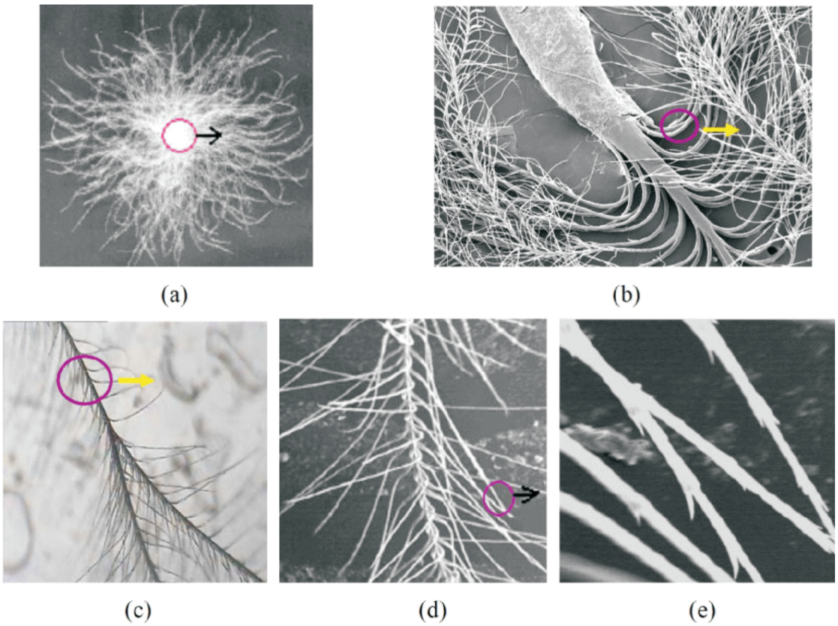


**Figure 1** Local, statistical self-similarity of goose down fiber. (a) Individual down cluster. (b) Down branches diverging from down nucleus in down cluster. (c) Secondary branch diverging from main branch. (d) Down fibrils diverging from down branch. (e) Crotch nodes on down fibril. The magnifications of (a), (b), (c), (d), and (e) are 1, 120, 20, 100, 950 times, respectively.

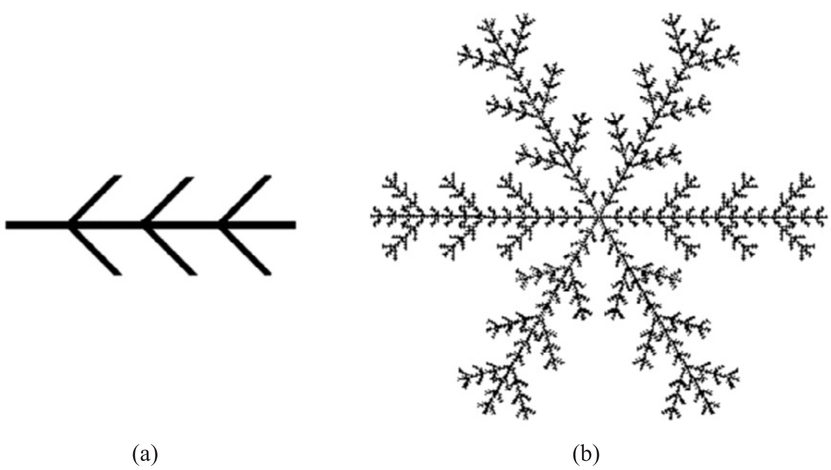


**Figure 2** The basal generating unit (a). Computer generated fractal graphics at three levels of hierarchic iterations (b).

unit in Figure 2(a) for three hierarchical levels shown in Figure 2(b). According to the concept of fractal dimension in a global sense, the unit consists of 10 similar to a quarter of the whole, so the fractal dimension of goose down can be calculated using

$$d_f = \frac{\ln N(L)}{\ln L} = \frac{\ln 10}{\ln 4} = 1.66 \qquad (2)$$

## Actual Evaluation of $d_f$ Based on a "Sandbox" Method

In order to evaluate the actual "local fractal dimensions" of goose down [16], we first need to select a representative image of down and then deal with the image to adapt the computer calculation programs.

We selected the image shown in Figure 3 as the initial structural unit for analysis. To obtain the best imaging process effect, we need to determine a critical pixel level $p_{cr}$, which will enable us to differentiate the actual structural information from the background noise. In other words the critical pixels level $p_{cr}$ ensures a close rendering of the actual picture into a computer image with the least distortion. It is a monochromic case so that we use the gray level to regenerate the image. By choosing the background as total black and the goose down as total white, Figure 4 provides the information between the pixels numbers versus the gray levels from the initial image. It is not difficult

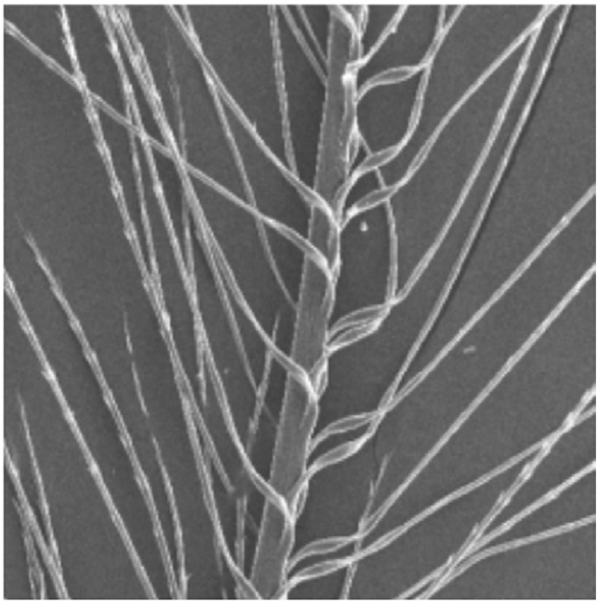

**Figure 3** Initial structural unit image.

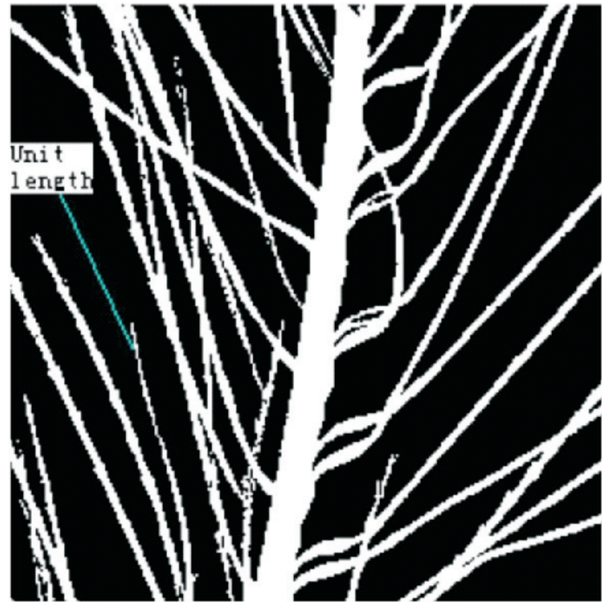


**Figure 5** The picture after binary processing.

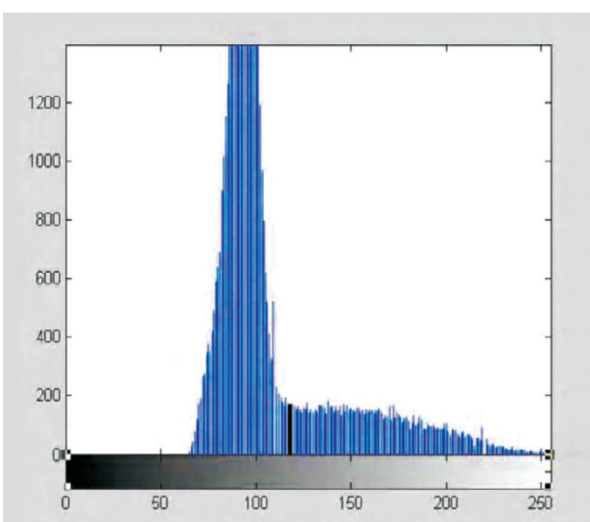


**Figure 4** Selection of the critical pixels.

to discern that from the left to the right of the figure, as the gray level changes from darker to whiter, the corresponding pixel number is reduced. It is at the pixel number $p$ = 117 where the profile shows a turning point; that is, by choosing $p_{cr}$ = 117, the contrast between the image and background is at its highest.

Then by using a two-value algorithm based on the threshold $p_{cr}$ = 117, we can turn all of the pixel values into a binary system of black and white:

$$p \geq p_{cr}, \text{ gray level} = 1, \text{ black}$$

$$p < p_{cr}, \text{ gray level} = 0, \text{ white}$$

A goose down image thus enhanced is shown in Figure 4.

To measure the local fractal dimension of Figure 5, we adopt the *Sandbox* methods of fractal theory, proposed by Stanley et al. in 1985 [15].

First, we need to identify the appropriate range of length scales $L$. Since we are interested even in the finest of fibrils or nodes on the down, clearly the length scale $L$ should be the finest fiber diameter or even smaller than that. For various areas (black or white), calculate the square box numbers $N(L)$ at a corresponding given size $L$. Then the slope of a log–log plot of $N(L)$ against $L$ will be defined as the fractal dimension value $d_f$ of the goose down fibers.

As a demonstration, we show how this calculation is conducted using Figure 5. Select the finest branch diameter of the goose down as the basic unit of the measure scale $l$, and the side length of this figure as $l'$, so

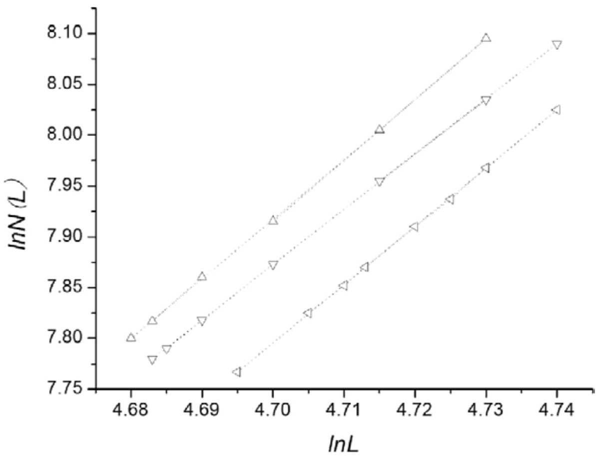


**Figure 6** Log-log plot of *N(L)* against *L*.

$$L = \frac{l'}{l} = \frac{230}{2} = 115 \quad (3)$$

Using an area calculating procedure and noting that there are a total of 255 pixels for each side, we can obtain the areas occupied by white pixels and black pixels, respectively. The total white pixel occupancy is 24.55%, so

$$100\%:\frac{l'^2}{l^2} = 24.55\%:N$$

that is

$$N = 0.2455 \times 230^2/2^2 = 3246.7375 \quad (4)$$

Thus, the fractal dimension is

$$d_f = \frac{\ln N}{\ln L} = \frac{\ln 3246.7375}{\ln 115} = 1.704 \quad (5)$$

By assigning different $L$ values and calculating the relevant $N(L)$ values, we can plot a log–log curve as in Figure 6. The curve of the double logarithm has shown remarkable linearity, and the linearity correlation coefficient achieves 0.99, clearly revealing the prominent fractal characteristics of the goose down fibers. Thus the fractal dimension $d_f$ of the system can be derived through equation (2).

Since this is an experimental approach with inevitable statistical error, at different measure scales, the respective fractal dimension $d_f$ will be slightly different. A statistical estimate based on adequate data can lead to a more accurate value of $d_f$.

It is both surprising and understandable that after many generations of evolution, goose down has acquired such an interesting fractal dimensional structure. The fractal dimension of the goose down fiber is calculated as 1.66 by computer simulations and 1.704 by actual measurement of Sandbox methods, close to the golden mean, 1.618, which assuredly reveals certain structural optimality in goose down. In the known research of the mathematical aspects, Iovane [8] showed the importance of the golden mean with respect to the large-scale structures in our universe; Weiss [16] illustrated that the principle of information coding by the brain seems to be based on the golden mean. Keshavarzi et al. [17] found that the fractal dimension of turbulence is relative to 1.618; He [18] applied the golden mean to biomechanics. The main application of golden mean in E-infinity theory [19, 20] shows miraculous exactness compared with experimental measurement, especially in determining the theoretical coupling constants and the mass spectrum of the standard model of elementary particles. A new mathematical direction called harmony mathematics based on the golden mean has also been systematically established. So we can be sure of the significance of the value of $d_f$ for the survival of geese is quite self-explanatory, and the optimal structure characteristics of single down fibers contribute to the thermal insulation of assemblies.

## Conclusions

Using fractal methods, studies of the morphological structure of goose down have shown that down fibers have typical local fractal "tree" structures. The many different grades divaricators in goose down have obvious local, statistical self-similarity with both each other and the whole.

The fractal dimension $d_f$ is calculated as 1.66 by computer simulation and 1.704 by actual measurement of using a "Sandbox" method, a value close to the golden mean, 1.618, confirming the inherent optimal structure of goose down. What is more, in the two-dimensional plane, when $1 < d_f < 2$, it can be assumed that the curve has infinite length as well as a very limited area. As $d_f$ approaches two, the fibers tend to occupy more "space" and limited autovolume. So the specific areas of fibers become larger and larger, and the fibers dichotomous structures become increasingly evident. According to theory of fluid mechanics, system permeability is in inverse proportion to the square of medium specific areas. Thus, it can be seen that airflow circulation resistance will grow remarkably with the increasing of fiber-specific area. All of these factors make down fiber assembly retain a mass of immobile air and thus keep superior thermal insulating properties. The further mechanisms associated with this, however, are still under investigation in our ongoing research.

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