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Modelling Fabric's Optical Behaviors

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Abstract. In this paper, existing models of fabric's optical behaviors are classified and estimated briefly. Then, basing on the optical theories, this paper abstracts the commonness of fabric's optical behaviors and put forward an improved model of them, which includes a physical model and its corresponding mathematical model. The physical model describes the light falling on fabrics splits into six components, which compose another four components. The ten light components divided by the incident light yields ten light component parameters, which are functions of the wavelength. The mathematical model connects the ten light component parameters and gives formulas to calculate them. Thus, the light component parameters are connected with the basic optical parameters and structural parameters of fabric. This improved model provides a fundamental and theoretical guide for studying fabric's optical properties systematically, which has been proved to be effective in solving some problems related fabric optical properties.

Introduction

Fabric's optical behaviors, which include ultraviolet blocking property, color, lustre, transparency, solar-optical properties, infrared properties, military camouflage and so on, relate to not only aesthetic, serviceability, but also protection, health care and energy-saving. They play increasing important roles in modern life and modern military with the increasing demands for both fashion and health care, with the continuous decrease of energy resource and with the continual regeneration of military detective apparatus.

Researches on fabric optical behaviors mainly focused on an individual behavior in a narrower waveband such as in ultraviolet or visual or infrared region. There are few systematical and theoretical studies focusing on the commonness of fabric optical behaviors. However, in practical, several fabric optical properties in the whole light band are usually required at the same time.

As we know, although fabrics' optical properties are diversified, all of them are the responses of fabrics to electromagnetic waves. Hence, basing on the basic optical theories, this paper tries to establish a more universal model for fabric optical behaviors by abstracting their commonness. This model can be adopted as a theoretical guide to better understanding and systematically studying optical behaviors of fabric.

Classifying the Existing Models

In order to summary the commonness, models for fabric's optical behaviors are classified into physical and mathematical ones. The existing physical models are classified into three-component model [1], four-component model [2], and other models [3]. Three-component model is the conventional one, in which, a light falling on fabric was divided into three parts according to the resulting light distribution, i.e., reflection, absorption and transmission. Four-component model was derived from a more detailed division by subdividing the transmission part in three-component model into a component that passes through the pores and a diffused or scattered one (see Fig.1).

The existing mathematical models are classified into light fluxes models [4,5,6], fibers pileup models [7,8], fibers pileup-pores models [9,10] and ray trace models [11,12]. Among which, the two fluxes Kubelka-Munk model [4] has been most widely used, especially in the field of color technology.

These existing models were originally set up for certain fabric optical behaviors. They are useful in evaluating certain optical properties of fabrics, but not much so when used to analyze the reasons for fabric's optical properties. Furthermore, some of the models misunderstood the scattering coefficients and misused some optical laws.

An Improved Model for Optical Behaviors of Fabric

Basing on the interaction between light and fabric, by abstracting the commonness of fabric's optical behaviors, the paper puts forward an improved model, which is described by an improved physical model and its corresponding mathematical model.

Improved Physical Model. The improved physical model describes the light illuminating on fabric splitting into six components, which compose another four components. The ten light components divide by the incident light yields ten light component parameters.

In examining the interactions between light and fabrics, the incident light illuminating a fabric is divided as follows (see Fig.2): part of the light turns into regular reflection, including mirror/specular reflection and diffused reflection at the interface, marked as I_{mR} and I_{dR} respectively; another part I_A enters into the fabric and is absorbed by the material; still another part enters into the fabric and is scattered due to the uneven structure of the fabric and/or the additives in the fabric, and the left passes through the fabric with direction unchanged, recorded as I_{dT}. Of the scattered light, the back-scattered light generates the same effect as the reflection, termed scattered reflection I_{sR}, whereas the forward-scattered light also passes through the fabric, portions maintaining the same direction as that of the incident direction are recorded into I_{dT}, the rest deviating from the incident direction are terms as scattered (or diffused) transmission I_{sT}. In short, the incident ray is eventually transferred into six portions, as following: mirror (or specular) reflection I_{mR}, diffused reflection I_{dR}, scattered reflection I_{sR}, absorption I_A, direct transmission I_{dT}, and scattered (or diffused) transmission I_{sT}. The six portions compose another four components, i.e. (hemispherical) reflection I_R, regular reflection I_{rR}, hemispherical transmission I_T and scattering I_s. Their relations are list in Eq. 1.

$$
\begin{aligned}
&I_{mR} + I_{dR} + I_{sR} + I_A + I_{dT} + I_{sT}=I_0 \\
&I_{mR} + I_{dR} + I_{sR}=I_R \\
&I_{dT} + I_{sT}=I_T \\
&I_{mR} + I_{dR}=I_{rR} \\
&I_{sR} + I_{sT}\approx I_s\,.
\end{aligned}
\qquad (1)
$$

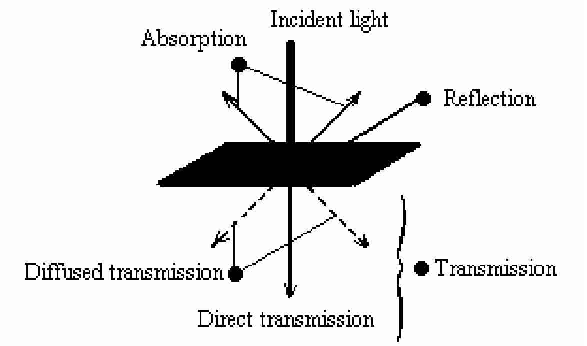

Fig.1 Conventional four-component model

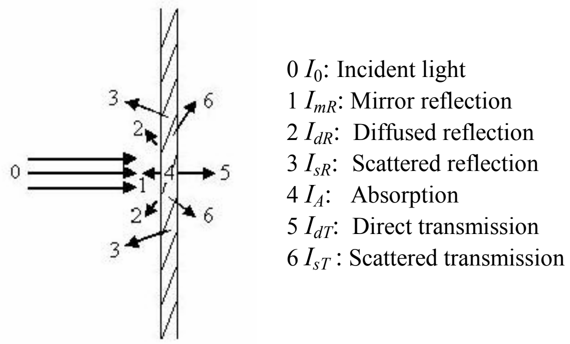

Fig.2 Improved physical model

The intensity of the ten portions divided by that of the incident light yield ten light component parameters, which are functions of wavelength. Their relationship is shown in Fig.3. Besides, there is a special parameter, R_∞, the hemispherical transmission of infinite thickness of materials, which is applied as the substitute for R in some cases.

Thus every optical property of fabrics can be characterized by one of them or a combined parameter from some of them, or a transformed parameter from them. For instance, the UV-Blocking property can be characterized as the function of T in the ultraviolet band.

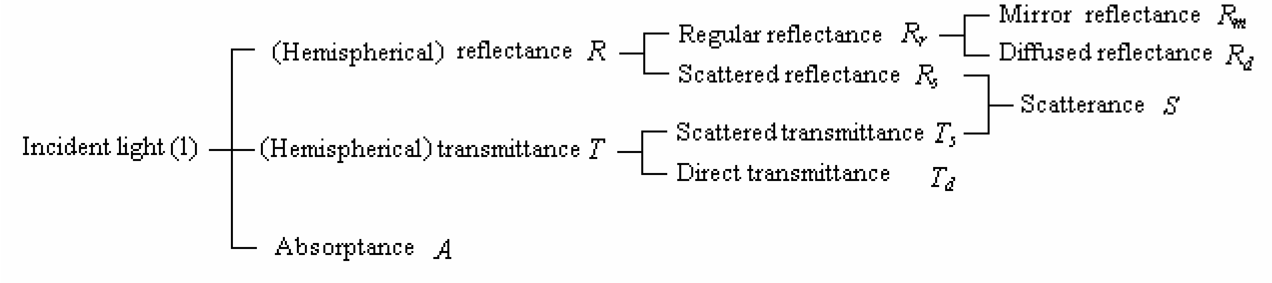

Fig. 3 Physical parameters in the improved model

It can be seen that the improved scheme adds scattered light, which is then subdivided into scattered reflection, direct transmission and diffused transmission according to their final effects. It seems that the improved modal is only more detailed than the conventional one. But, in fact, since the improved one is based on the interaction of light and fabric, it provides more initial information, so that it is more practical. What's more, scattering is an indispensable part of light interactions with turbid-medium; it does play an important role in determining fabric's optical behavior. Compared with the conventional one, the improved model are more practical in characterizing and analyzing fabric's optical properties essentially, meanwhile, it has great advantages in exploring the possible approaches to improve them.

Improved Mathematical Model. The improved mathematical model connects the ten light component parameters and deduces formulas to calculate them.
According to the improved physical model, following relations hold

$$R+A+T=1, \quad R_r+R_s=R, \quad R_m+R_d=R_r, \quad T_d+T_s=T, \quad R_s+T_s\approx S. \qquad (2)$$

These ten parameters are classified into four groups. (1) R_m, R_d and R_r, which follow the laws of reflection. (2) R_s, T_s and S, which follow the laws of scattering. (3) T_d. (4) R, T and A. The degree of freedom of the ten parameters is five. That is, if we know five independent parameters of them, we will get all the ten parameters.

Calculation of *T* and *R*. As we known, fabric is a kind of turbid material. Assume that the light fluxes in fabrics are similar to that described in Kubelka-Munk model [4]. Accordingly, the constitutive equations are listed as Eq. 3, the same to that in Kubelka-Munk theory.

$$\begin{cases} dj = -(K+S)j\cdot dx + Si\cdot dx \\ -di = -(K+S)i\cdot dx + Sj\cdot dx \end{cases}. \qquad (3)$$

Solve the differential equations, we obtain

$$\begin{cases} i(x) = c_1 e^{\phi} + c_2 e^{-\phi} \\ j(x) = c_1(a-b)e^{\phi} + c_2(a+b)e^{-\phi} \end{cases}. \qquad (4)$$

Where K and S are Kubelka-Munk absorption coefficient and scattering coefficient respectively, and $a \equiv (K+S)/S$, $b=\sqrt{a^2-1}$, c_1 and c_2 are two constants, $\phi = bSx$, $\Phi = bSX$, X is the thickness of the fabric.
According to the improved physical model, the boundary conditions are $i=1-R_r=R_s|_{x=X}$, $j=0|_{x=0}$ (Without background reflection), which are different from that in Kubelka-Munk model. In Kubelka-Munk model, surface reflection is ignored and back ground reflectance R_g is taken into consideration, therefore, its boundary conditions are $i=1|_{x=X}$, $j=R_g|_{x=0}$.
Insert the boundary conditions of the improved model into Eq. 4, we get Eq. 5

$$\begin{cases} i(x) = \dfrac{a\sinh\phi + b\cosh\phi}{a\sinh\Phi + b\cosh\Phi}(1-R_r) \\ j(x) = \dfrac{\sinh\phi}{a\sinh\Phi + b\cosh\Phi}(1-R_r) \end{cases} \quad (5)$$

According to the improved physical model, $T=i|_{x=0}$, $R_s=j|_{x=X}$, $R= R_s+R_r$, hence,

$$T = \frac{b}{a\sinh\Phi + b\cosh\Phi}(1-R_r) = T_i(1-R_r) \quad (6a)$$

$$R_s = \frac{\sinh\Phi}{a\sinh\Phi + b\cosh\Phi}(1-R_r) = R_{0i}(1-R_r) \quad (6b)$$

$$R = \frac{\sinh\Phi}{a\sinh\Phi + b\cosh\Phi}(1-R_r) + R_r = R_{0i}(1-R_r) + R_r = R_{0i} + R_r(1-R_{0i}) \quad (6c)$$

Increasing the thickness X to a certain degree, then $\cosh\Phi/\sinh\Phi=\coth(bSX)\rightarrow 1$, R_∞ is solved as

$$R_\infty = \left(1+\frac{K}{S}-\sqrt{(\frac{K}{S})^2+2\frac{K}{S}}\right)(1-R_r)+R_r = R_{\infty i}(1-R_r)+R_r \quad (6d)$$

In Eq. 6, the T_i、 R_{0i} and $R_{\infty i}$ are equal to the T, R_0 and R_∞ in Kubelka-Munk model respectively. From Eq. 6, we can deduce the correction equations for Kubelka-Munk model used for fabric, see Eq. 7.

$$T_i = \frac{T}{1-R_r}, \qquad R_{0i} = \frac{R-R_r}{1-R_r}, \qquad R_{\infty i} = \frac{R_\infty - R_r}{1-R_r}. \quad (7)$$

According to the above equations and Kubelka-Munk theory, several practical methods can be adopted to calculate the Kubelka-Munk absorption coefficient K, the Kubelka-Munk scattering coefficient S, and K/S.

(1) Measure T and R, calculate T_i and R_{0i} by Eq. 7, then K and S can be obtained by Eq. 8a

$$a = \frac{1+R_{0i}^2 - T_i^2}{2R_{0i}}, \qquad S = \frac{1}{Xb}\ln\frac{1-R_{0i}/(a+b)}{T_i}, \qquad K = S(a-1). \quad (8a)$$

(2) Measure R and R_∞, calculate R_{0i}和$R_{\infty i}$ by Eq. 7, then K and S can be calculated by Eq. 8b

$$a = \frac{1}{2}\left(\frac{1}{R_{\infty i}}+1\right), \qquad S = \frac{1}{Xb}Ar\cot\frac{1-aR_{0i}}{bR_{0i}}, \qquad K = S(a-1). \quad (8b)$$

(3) Measure T and R_∞, calculate T_i and $R_{\infty i}$ by Eq. 7, then K and S can be attained by Eq. 8c

$$a = \frac{1}{2}\left(\frac{1}{R_{\infty i}}+1\right), \qquad S = \frac{1}{Xb}(Ar\sinh\frac{b}{T_i}+\ln R_{\infty i}), \qquad K = S(a-1). \quad (8c)$$

(4) Measure R_∞, calculate $R_{\infty i}$ by Eq. 7, then K/S can be gained by Eq. 8d, or directly calculate K/S by Eq. 8e, which coincides with one of Pineo's equations of surface reflection correction for K/S [13].

$$\frac{K}{S} = \frac{(1-R_{\infty i})^2}{2R_{\infty i}}, \quad (8d)$$

$$\frac{K}{S} = \frac{(1-R_\infty)^2}{2(1-R_r)(R_\infty - R_r)}. \quad (8e)$$

Calculation of T_d. In compliance with the improved physical model and extended Lambert law, T_d can be calculated by Eq. 9

$$T_d = \exp[-(a+s)X](1-R_r) = T_{di}(1-R_r) \quad (9)$$

While, a is absorption coefficient, s is total scattering coefficient, a+s is the extinction coefficient, T_{di} is the transmittance in Lambert law.

Solution of R_r and R_d. Before solving the R_r of fabric, let's look into $R_{r\text{-}f}$, the regular reflectance of fibers. For homogeneous material, according to reflection law and Fresnel Formula, its regular reflectance of normal incidence is displayed in Eq. 10

$$R_{r\_m} = \frac{(n-1)^2+k^2}{(n+1)^2+k^2} \quad (10)$$

Where n stands for refractive index and k is a dimensionless absorption coefficient. $k=a\lambda/4\pi$, λ is wavelenghen and a is the absorption coefficient in Lambert law. For ordinary fiber, the refractive index n_f is about 1.4~1.7, and k is far less than n, almost zero in visible light. If we assign n_f=1.55(n of cotton fiber) , then $R_{r\text{-}f}\approx$0.046. For fabric,

$$R_r=h\varepsilon R_{r\_f}. \quad (11)$$

Where ε represents the cover coefficient of fabric, h is a structural factor, ranging from 0 to 1.

For ordinary fabric, it is very difficult to calculate the precise values of R_r, R_m or R_d, because the structure and surface of fabrics are too complex to be described exactly, especially for those made of staple fibers. McLaren[14] provided an experimental solution to R_r: dye fabric with yellow colorant by stepped concentrations, measure their reflectance R, when the value of R at absorption peak is no longer decreasing with the increase of concentration, the R at the absorbing peak is the R_r of the fabric. Accordingly, the reflectance excluding mirror reflection at the absorption peak is the R_d of the fabric. We has used McLaren's method to determine the reflectance of a white cotton drill, 21×21 108×58, R_r =0.02. Then we get $h\approx$0.5 by Eq. 11.

Summary of the Improved Model. As described above, we derived five independent light component parameters of the improved model for fabric's optical behavoirs, the other five parameters can be calculated by Eq. 2. On the other hand, Eq. 6~Eq. 11 build the bridge between basic optical parameters, basic structural parameters and light component parameters of fabric. In this way, all the parameters can be solved.

Application of the Improved Model

The improved model has been applied to analyzing fabric's optical phenomena. For example, we applied it to study a debated issue, the mechanisms of inorganic semiconductor oxide as ultraviolet-blocking additives for fabric. Our research report was quickly accepted and published [15]. Except the actual mechanism, many other interesting and useful results are achieved, which are of great help for developing both ultraviolet-blocking additives and ultraviolet-blocking textiles. In addition, the improved model shows more advantages in characterizing the optical properties of fabric. Moreover, possible approaches to improve fabric's optical behaviors can be obtained by analyzing the factors and principles of changing the light component parameters. The improved model has also been applied to systematically analyze Kubelka-Munk single constant / two constants theory, the basic principle of color matching, on which a paper has been organizing.

Summary

Basing on optical theories, this paper systematically studied the model for fabric's optical behaviors. First, the existing models are classified and evaluated. Secondly, this paper develops an improved model, an integrated one of physical model and mathematical model. The improved model has been applied to solve practical problems and has been proved to be more robust. The improved model has shown its advantages in both characterizing and analyzing fabric optical behaviors, and in investigating possible approaches to optimizing fabric optical behaviors, as well as in studying some other related issues. It provides a fundamental and theoretical guide for systematically studying optical behaviors of fabrics.

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