Fractal Dimension of Down Fibre Assemblies

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Abstract. As porous media materials, down fiber assemblies have excellent heat insulating and are widely used in the thermal products. The internal microstructures and fibers arrangement strongly influence heat conduction in down fiber assembly. This work presents a unified treatment using the tool of "local fractal dimensions" to describe the geometric complexity of the relative fibers arrangement in the down fiber assembly.

1. Introduction

Down fibers are a layer covered on the cuticular of fowl, such as goose, duck and swan. Down fiber assemblies are composed by a mass of down fibers disordered arrangement. They belong to typical kind of porous materials and have excellent thermal insulation properties as the filler materials of thermal products in winter. In the fibers assembly, the fibers arrangement and the pores distribution influence heat conduction strongly. Traditional approaches to engineering studies describe the porous medias of fiber assemblies as "average volume", which suppose the fibers ordered arrays. However, in the down fiber assemblies found in practice, for example shown in figure 1, the fibers hardly assume an ordered arrangement. So the assumptions don't agree with practical situations. What's more, the ordered array simplification is inaccurate in describing such porous medias in almost all practical situations. From a practical point of view, it is desirable to have a universal means of quantifying the relative fiber arrangements, which will be applicable to disordered arrays. In response to this, this work introduce a method, wherein the pattern of fiber arrangement in a cross section is characterized as a local fractal object, and described in terms of local fractal dimensions. The local fractal dimensions of a fiber assembly cross section are the fractal dimensions that it exhibits over a certain small range of length scales.

Before presenting the method, an approach to observe the fiber assemblies microstructure is briefly introduced in the following section.

2. Micro-CT method to assemblies microstructure observing

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Down fiber assembly is one kind of low-density loose assemblies, and the internal structures are easy to be destroyed by general observing such as slitting or embedding. If we adopt container filled with fibers observing in terms of certain volume fraction, the shown images (as figure 1) obviously can't reflect single layer fibers arrangement, thereby it is not an effective method too. In response to this, we employ Micro computed tomography (Micro-CT) [3] in the study of down fiber assemblies microstructure. Predominance of Micro-CT exhibits its higher resolution power and none influences on materials structures and shapes.

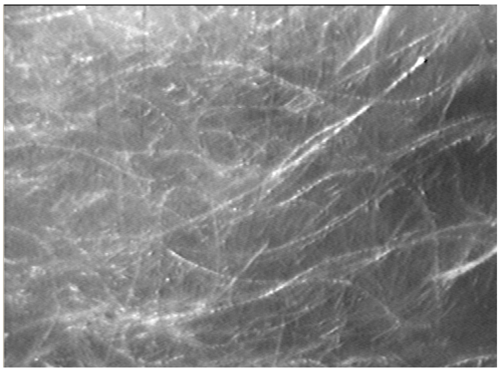


Figure 1. General observing of down fiber assembly

Use SkyScan-1172 High-resolution desk-top micro-CT system made in SkyScan Ltd. of Belgium to observe the microstructures of down fiber assemblies. Firstly, fill the down fibers of various volume fractions in transparent column vessels made of polyester film, as shown in figure 2. Then scan the vessels cross sections in this micro-CT system. So that the fibers disordered arrangement and the microstructure of the fiber assemblies can be looked into at nano levels. The volume fractions (Vf) of the fiber assemblies are selected respectively as: a. V_f=0.001162, b. V_f=0.001960, c. V_f=0.005659.

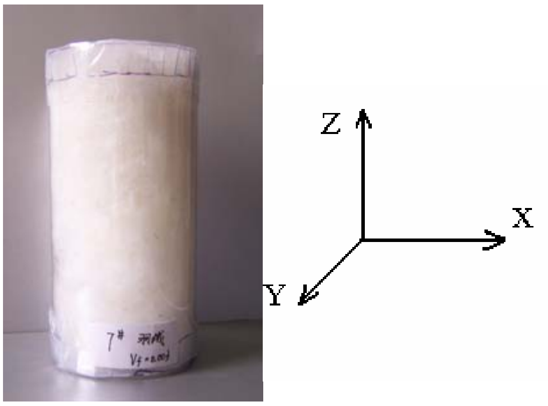


Figure 2. Down fibers sample in column vessel

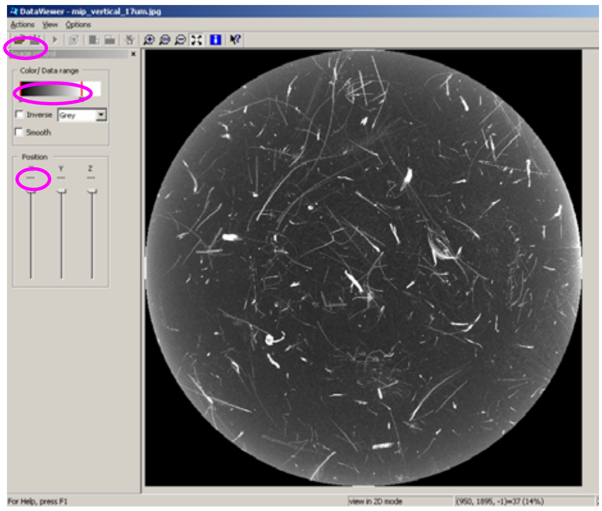


Figure 3. Observing screen of Micro-CT

Figure 3 has shown Micro-CT observing screen of down assembly. The top left corner on the screen displays various function bonds to control the observing patterns and angles to assembly. Especially, adjusting the "Position" bond can change the images of various positions along direction X, Y or Z. Unite the many images in various directions, they reflect the arrangement and action modes among fibers.

From figure 3 we can see that the distributions of fibers and pores in fiber assemblies are inconsistent evidently. There is a kind of complicated micro-space structure. The internal structures vary with the fiber diameters, fiber lengths and fibers arrangement. The fibers accumulate types and the pores shapes are both random and irregular. Euclidean geometry describes the regular objects of macroscopical size using integer dimensions 0, 1, 2 and 3 respectively. However, it can not describe expressly the complexity of internal micro-space structure of porous media like fiber assemblies. The fractal geometry just happen to solute the problems using fractal dimension. In order to quantify the complexity of fibers arrangement in assemblies, we employ the concept of “Local fractal dimension”, which will be described in the following section.

3. Local fractal method description

A fiber assembly cross section, although it may not be an exact fractal by nature, exhibits statistical self-similarity over a small range of length scales around a fiber. It is observed that the average fiber area, *N(L)*, enclosed within squares of different sizes *L* by *L*, scales with L in accordance with the equation (1) over a range of length scales spanning a few fiber diameters. Because of the small range of length scales, the use of the term “local fractal” seems appropriate, although the present range is much smaller than those in the conventional fractal applications.

$$N(L) \sim L^{d_f} \tag{1}$$

The following procedure for the evaluation is a variation of the “Sandbox method” of measuring local fractal dimension (figure 4), proposed by Stanley [3] in 1985:

(1) Discretize the fiber assembly cross section into a number of grid cells so as to ensure equiprobable samplings in the procedure described below. Typically, grid cells about one-tenth the fiber diameter in size provide a good resolution of the cross section.

(2) By placing a line of length L parallel to the direction of heat flow on every grid cell, systematically scan the entire cross section. This is equivalent to a purely random sampling with every cell being an equally probable origin for the measurement. Each time the center of the line falls on a fibrous region, record the total length of the intercepts between the line and the fibers. The average of these intercepted total lengths is the mean length N(L) .

(3) Considering the heat transfer of fibers is related only to the neighboring fibers, choose d≤L≤d+l as the measure range. Repeat step (2) above for several values of L within the measure range, and make a log-log plot of N(L) against L.

(4) The slope of the best line fit through the points in this plot gives the exponent in Equation(1), which is the local fractal dimension df.

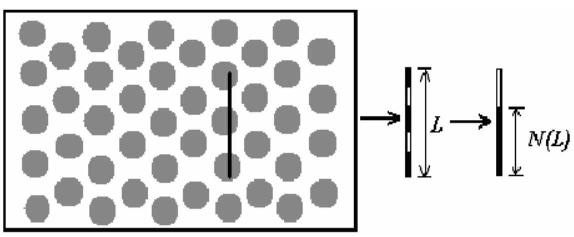
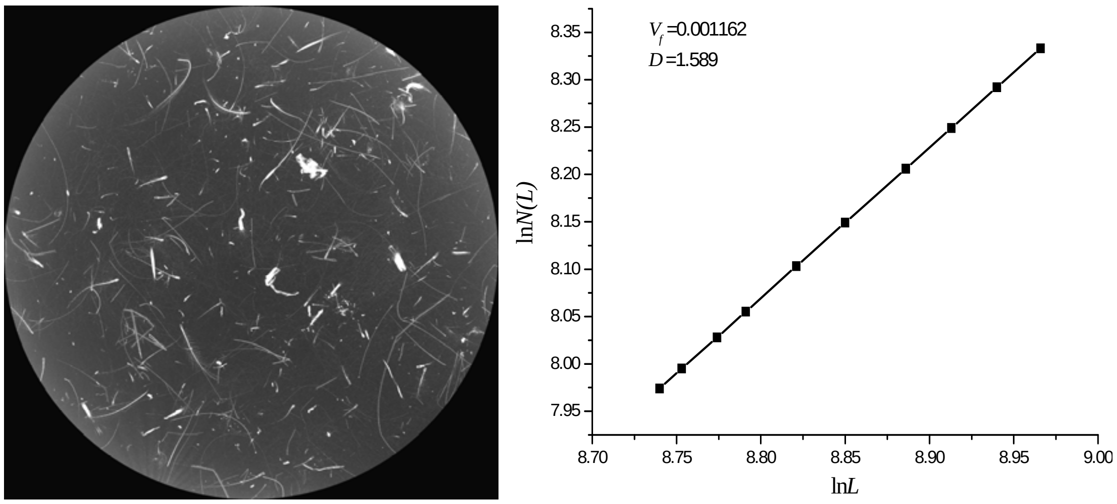


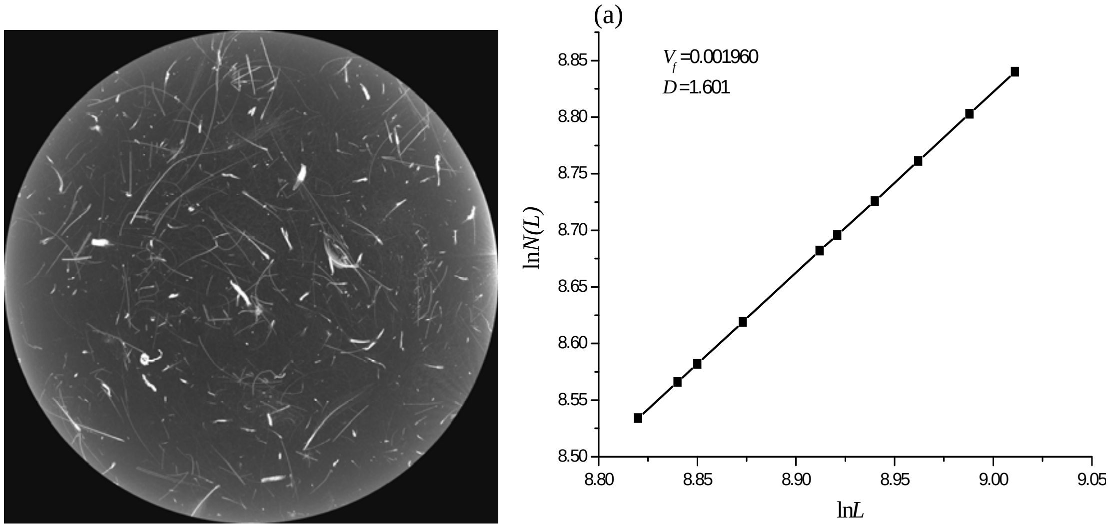
Figure 4. Sketch map of Sandbox method

4. Local fractal dimensions evaluation of down fiber assemblies

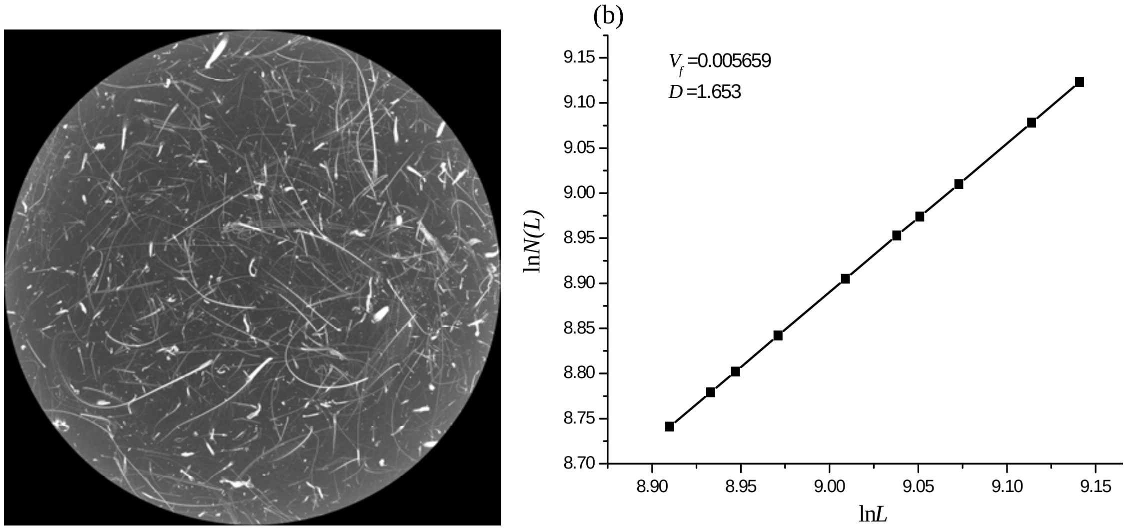
Figure 5 has showed down assemblies the cross sections images with V_f =0.001162, 0.001960 and 0.005659 respectively. Wherein the white parts represent the down fibers, and the black parts represent pores. As we can see that, as the fiber volume fraction increasing, the fibers are packed more closely, and the distributions of both fibers and pores are more complicated.



(a)



(b)



(c)

Figure 5. Fiber assemblies images (on the left) and the corresponding Log-log plots of N(L) against L (on the right). (a) V_f=0.001162, (b) V_f=0.001960 and (c) V_f=0.005659.

Selecting the box side 2μm and 30μm＜L＜100μm, employ the "Sandbox" method introduced above to perform the images respectively. The log-log plots of N(L) against L can be obtained as figure 5 shown. The figures reflect excellent line fits in all three situations. The linearity equations are following respectively:

$$\textbf{a.}\ \ln(N(L)) = -5.91 + 1.589\ln(L)$$

$$\textbf{b.}\ \ln(N(L)) = -5.59 + 1.601\ln(L) \qquad \textbf{(2)}$$

$$\textbf{c.}\ \ln(N(L)) = -5.59 + 1.601\ln(L)$$

All of linearity relativities reach 0.99. Then the local fractal dimensions can be calculated as 1.589, 1.601 and 1.653, which explain the obvious fractal ability of fibers arrangement in cross section of down assemblies.

Also we can draw the conclusions from the results. A closely packed array, resulting from a high fiber volume fraction, corresponds to a high degree of "connectedness" of fibers. Consequently, its local fractal dimensions will be close to 2. Conversely, the farther away the fibers are relative to one another, resulting from a low fiber volume fraction, the smaller the value of the local fractal dimensions is.

5. Conclusions

Down fiber assemblies are typical kinds of porous media materials. In the porous media materials, the fibers arrangement and pores distributions are not ordered. The practice is not consistent with the general suppose in engineering studies. In order to quantify the relative fiber arrangements and describe the assemblies' microstructure, we introduce a universal means of "local fractal dimension" to resolve the fractal aperture problems in general porous materials. Through Micro-CT method, the cross sections of fiber assemblies are looked into in various positions. Employ "Sandbox" method to perform the images, the local fractal dimensions of cross sections in various volume fractions could be calculated respectively. The results indicate that the down fiber assemblies as porous materials exhibits statistical self-similarity and have typical local fractal characters over a certain small range of length scales. To a high fiber volume fraction, its local fractal dimensions will be close to 2. Conversely, a lower fiber volume fraction the assembly is, the smaller the value of the local fractal dimensions.

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