Fibers and Polymers 2007, Vol.8, No.4, 399-407

Extensible Elastica Solutions on the Large Deflection of Fiber Cantilever with Circular Wavy Crimp. II. Classification of Equilibrium Configurations

Jae Ho Jung, Ning Pan[1], and Tae Jin Kang*

School of Materials Science and Engineering, Seoul National University, Seoul 151-742, Korea
[1]Division of Textile and Clothing, University of California, CA 95616, United States
(Received October 24, 2005; Revised June 28, 2007; Accepted July 2, 2007)

Abstract: The types of equilibrium shapes in the extensible elastica solutions of planar deflection of two-element crimped fiber cantilever with circular wavy crimp were classified for one end clamped boundary under concentrated, inclined and dead tip load. Material property of the fiber was also regarded linear elastic, while crimp was described as a combination of semicircular arcs smoothly connected with each other having constant curvature of all the same magnitude and alternative sign. Investigating the mathematical features of the previously obtained solutions shows that the possibility and variety of multiple equilibrium shapes originates from the following two reasons: the existence of Case 1/Case 2 type transformation, and the mathematical features of Case 1 type transformation. In two-element crimped fiber, there exist four different choices for Case1/Case2 type transformation. Regarding Case 2-Case 2 type equilibrium shape as the default and most probable one, other three types of equilibrium shape can be regarded as multiple equilibrium shapes. Moreover, Case 1 type transformation itself may lead to multiple configurations due to the possibility of the contra-flexure point after bending while Case 2 type transformation doesn't allow such characteristics. Case 1-Case 1 type equilibrium shape shows maximum varieties in shape, while Case 2-Case 1 and Case 1-Case 2 type shapes occasionally leads to the identical results with different choice of the sign. Tables and shapes in each configuration were also presented.

Keywords: Extensible elastica, Equilibrium configuration, Dead tip load, Crimped cantilever, Multiplicity of solutions

Introduction

In part I [1], we derived the extensible elastica solutions on the large deflection of fiber cantilever with circular wavy crimp under dead tip load, and drew the shapes of case 2-case 2 type equilibrium as the default and most probable one. As mentioned in our previous paper, they are merely one part of four possible choices of Case1/Case2 type transformation in each segment: Case 2-Case 2, Case 2-Case 1, Case 1-Case 2, and Case 1-Case 1.

It has been usually a common sense in mechanics that one loading condition renders only one possible, or unique, solution so long as all the others-material and geometrical properties, boundary conditions-do not change. Thousands of researches for last several decades, however, showed that there could be multiple solutions even in one loading condition. For example, Navaee and Elling [2] dealt with the equilibrium configuration of the bending of straight and inextensible cantilever beams.

Multiplicity of the solutions has an important meaning in that it implies we should consider the elastic stability of all possible equilibrium shapes. Most typical and well-known case is the buckling of straight column [3]. In viewpoint of mechanics the buckling itself is a shift of equilibrium shape from straight to non-straight configuration at a certain loading condition. Since the initial is also the equilibrium state after loading throughout the values of load, the non-straight configuration can be regarded as another possible solution. Of course, it depends upon the stability of the system whether the equilibrium chooses straight or non-straight shape. Here are two famous examples of stability analysis [4,5] on the buckling of incompressible/compressible straight column.

According to the same reason, the multiplicity of the solutions in bending deflection is deeply involved with its elastic stability. The only difference is that the initial shape is not equilibrium one after deflection, apart from the loading condition. In reference 3, one can find that even straight beam can show various equilibrium shapes at the same load. But there is no report about extensible elastica solution of crimped beam with inclined loading. As mentioned above, it has initially four choices of mathematical transformation, whereas the reference 3 can be classified as Case 1 type transformation only. Tables 2 and 3 in reference 2 make us glance at the diversity of equilibrium configurations, yet we do not know exactly how many configurations exist in our model. Thus the purpose of this paper is to classify it.

Summary of Previous Results

In our previous paper [1], we derived the following governing equation for two-element extensible elastica of crimped cantilever with circular cross-section. Nomenclatures are the same as those in the previous one.

$$\left(\frac{d\theta}{d\varepsilon_i}\right)^2 = \lambda_i(1-C_i)\left[1-\frac{2}{1-C_i}\sin^2\left\{\frac{1}{2}\left(\frac{\pi}{2}-\alpha+\theta\right)\right\}\right]\left[2-\mu C_i-\mu\left[1-2\sin^2\left\{\frac{1}{2}\left(\frac{\pi}{2}-\alpha+\theta\right)\right\}\right]\right] \quad (1)$$

*Corresponding author: taekang@snu.ac.kr

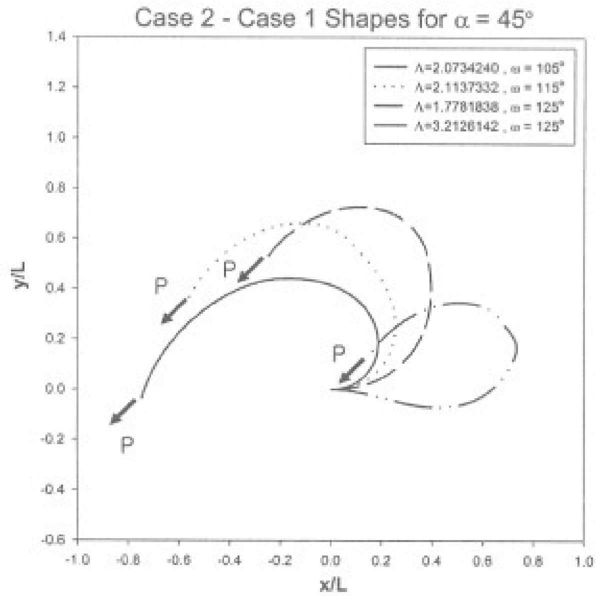

Figure 14. Shapes of case1-case 1 equilibrium configurations from the choice of the sign as minus-plus combinations with variable load for extensible elastica of r/ρ=0.1 at α=45°, ω=65°.

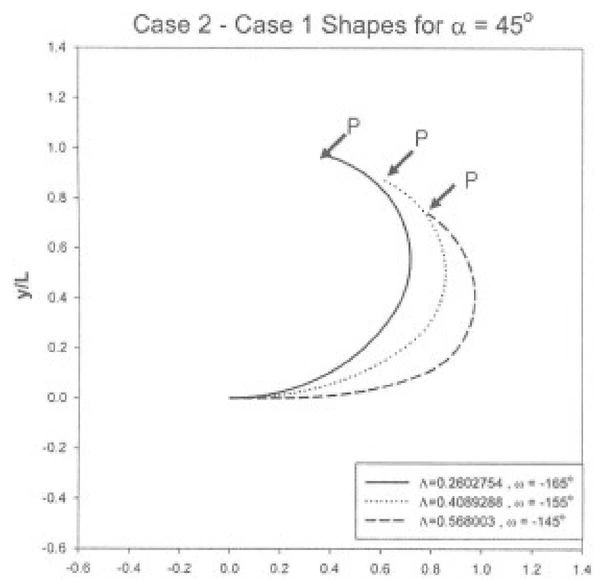

Figure 15. Shapes of case2-case 1 equilibrium configurations from the choice of the sign as minus-plus combinations with variable load and tip angle for extensible elastica of r/ρ=0.1 at α=45°.

Conclusion

Classification of the types of equilibrium shapes was made. Examples of circular cross-sectioned fiber of two elements whose radius ratio r/ρ is 0.1 with the range of $0 \le \Lambda \le 10$ at given inclined load angle α=45° were presented. As a result, many interesting types of the shapes have been obtained, which are very difficult to predict without modeling. Case 1-case 1 solutions have maximum variety of the types, whereas case 1-case 2 or case 2-case 1 types occasionally have the common shapes with different choices of the sign in the equation (9). From the shapes, it is concluded that the case 1 type transformation may have the inversion of the sign of curvature or contra-flexure point after bending, while case 2 type shapes show neither inversion nor contra-flexure point. It is very astounding feature that the equilibrium shapes of merely two-element crimped elastica show so various types of equilibrium shapes.

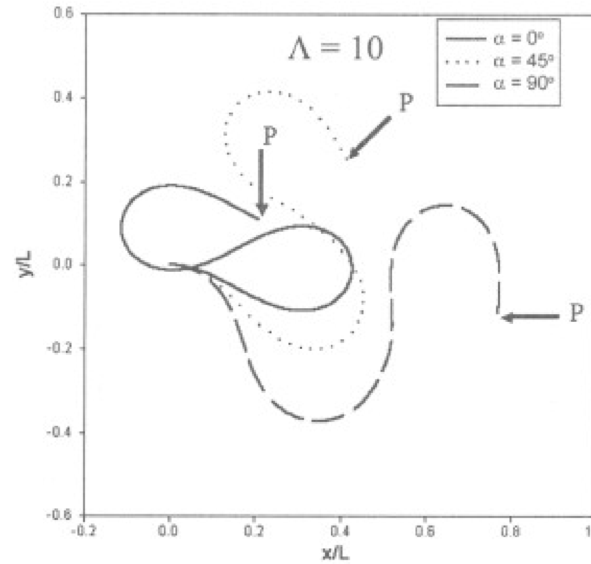

Figure 16. Example of the different shapes of case1-case 1 equilibrium configurations from the choice of the sign with variable inclined angle α=0°, 45°, 90° at Λ=10.

Acknowledgements

This work was supported by the SRC/ERC Program of MOST/KOSEF (R11-2005-065) through the Intelligent Textile System Research Center (iTRC) at Seoul National University. This work was also partially supported by National Textile Center(NTC) in University of California at Davis.

References

1. J. H. Jung and T. J. Kang, *Fibers and Polymers*, **6**(1), 55 (2005).
2. S. Navaee and R. E. Elling, *J. Appl. Mechan. Trans. ASME*, **59**, 572 (1992).
3. S. P. Timoshenko and J. M. Gere, "Theory of Elastic Stability", 2nd Ed., McGraw-Hill, 1961.
4. A. Magnusson, M. Ristinmaa, and C. Ljung, *Int. J. Solids Struct.*, **38**, 8441 (2001).
5. V. V. Kuznetsov and S. V. Levyakov, *Int. J. Non-Linear Mech.*, **37**, 1003 (2002).