The Elastic Constants of Randomly Oriented Fiber Composites: A New Approach to Prediction

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ABSTRACT

This article proposes a new and simple technique to predict the elastic material constants for a random fiber composite, including tensile and shear moduli and the Poisson's ratios for both 2-D and 3-D cases. Through a simple example how the Rule of Mixtures is derived for a unidirectional composite, the present author first demonstrates that what differentiates a random fiber composite from a unidirectional one lies in the fiber orientation, which can be best reflected from the relationship between the system fiber volume fraction V_f and the fiber area fraction A_f at a given direction of the composite. By establishing a general relationship between V_f and A_f using the fiber orientation density function, the author has developed a simple method based on the Rule of Mixtures to calculate the elastic properties of a random fiber composite.

The new model is compared to several existing models derived using more complicated mechanistic and mathematical theories. Previously published experimental data are also employed to verify the predictions of the new model.

The results are found to be reasonably satisfactory.

KEY WORDS: elastic properties, random fiber composites, fiber orientation density function, fiber volume and area fractions.

1. INTRODUCTION

Short fiber composites are being extensively used in various applications. The ease of manufacturing, good reinforced mechanical properties and macroscopic isotropy when fibers are randomly oriented are some of the unique advantages which make the short fiber composites an attractive category in composite materials. However, because of the complicated structural and mechanical features involved, short fiber composites remain one of the least understood areas in composite science /1/.

Fiber composites are non-uniform materials by definition: a fiber composite is a mixture of two distinct constituents, the fibers and the matrix material, with remarkably different properties. Another source yielding non-uniformities in a fiber composite is due to the intrinsic anisotropy of the fibers, assuming the matrix to be isotropic. Consequently, the properties of a composite are to a great extent dependent on the fiber orientations in the composite.

There have been numerous theories and models on the mechanics of short fiber composites, and some of the representative references are provided in the reference list of this paper /2-11/. The earlier ones are summarized in an excellent review article by Chamis and Sendeckyj /12/, and the more contemporary ones are collected and compared in /13/ by Chou. Recently, several new contributions to this area were made by Carman and Reifsnider /14/, Jasiuk, Chen and Thorpe /15/, Torquato and Lado /16/, Anlas and Stantare /17/, Eischen and Torquato /18/ and Giurgiutiu and Reifsnider /19/.

One of the major issues in short fiber composite study is to predict its elastic properties. The conventional method of calculating the moduli of a short fiber composite is to use the Halpin-Tsai equations /20/. Another rigorous approach dealing with the short fiber composite structure is based on the self-consistent method, which assumes that the fiber and matrix are isotropic, homogeneous and linearly elastic. Several authors /15,21/ have proposed theoretical models for short fiber composites using this technique. Still, a popular method is based on the classical laminate analogy /22-24/, where a random fiber composite is treated as a laminate constructed from a large number (> 3) or an infinite number of orthotropic plies oriented in all directions. This model, coupled with the usual micromechanic formulation, yields results for the properties of the composite. Also, several investigators /25-28/ have adopted Cox's technique /31/, utilizing the statistical density function to deal with fiber orientation in a random fiber composite so as to derive the elastic constants. Additionally, another alternative, named the bound approach, was also utilized by /16,18,32/ to predict the upper and lower bounds of the tensile modulus of short fiber composites. The problem with the existing techniques lies in the complexity of the results and the lengthy calculation procedures, and consequently these techniques are sometimes difficult to use in practice, unless simplified and thus losing their rigor.

A new and rather simple theoretical model is presented in this paper which is aimed to derive the relationship between the overall system fiber volume fraction V_f and the fiber area fraction A_f at a cross section of the composite. Combined with the Rule of Mixtures, this method is able to predict the elastic material constants, including various moduli and the Poisson's ratios.

It is widely recognized that accurate short fiber composite models are quite difficult to develop as a consequence of the complex interactions at the fiber-matrix interface as well as the limited fiber length and fiber ends effects. Therefore, a more or less idealized physical model becomes indispensable for any theoretical analysis. We hence adopt the following often used assumptions for the present analysis:

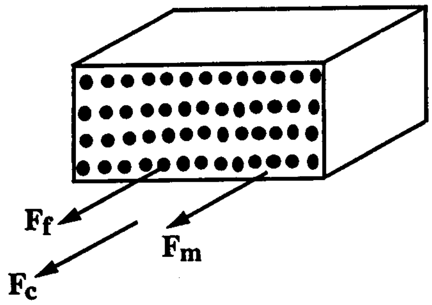
1. The composite consists of identical fibers, uniform in properties. Since the effect of fiber length has been investigated rather extensively /4,7/, and also this effect has little direct relevance to the new approach proposed below, we assume all the fibers are long enough so that this fiber length influence can be ignored.
2. All fibers are distributed uniformly along the length of the composite so that the fiber area fractions on all the cross sections in the same direction of the composite are identical.
3. Both fibers and matrix behave elastically. There is complete bonding at the interface of the constituents and the effect of the transitional region or interface on the elastic properties of the composite is excluded.
4. Furthermore, the fiber-fiber interaction within the composite and the effect of matrix property change as a result of the fiber interference are also ignored.

2. RELATIONSHIP BETWEEN FIBER VOLUME FRACTION V_f AND AREA FRACTION A_f

We will first show here that the critical obstacle which prevents the simple Rule of Mixtures from being applicable to a random fiber composite is due to the lack of a relationship between the system fiber volume fraction V_f and the fiber area A_f in a given direction. And this relationship is in fact dominated by the fiber orientation in the composite..

2.1. The unidirectional case

To elucidate the question clearly, let us go back to examine the simplest case: a unidirectional composite which is made of parallel fibers embedded in a matrix as depicted in Figure 1(a). When an external tensile load is exerted in the direction of the fiber axes which coincides with that of the composite principal direction, the strains experienced by the fiber, matrix and composite can be considered as equal



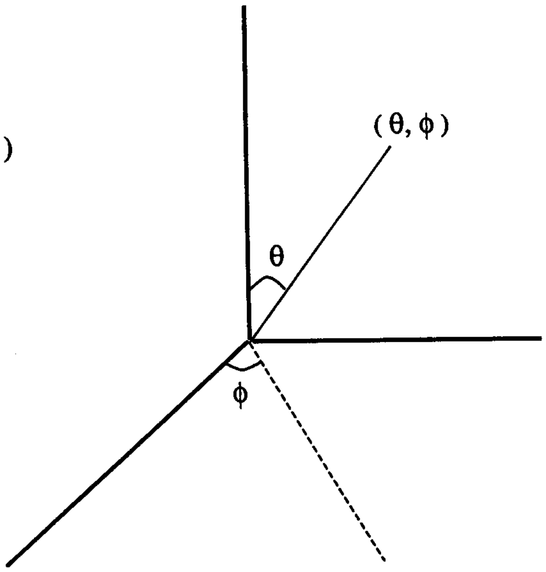


Fig. 1: A unidirectional composite and the coordinate system: (a) the forces on a cross section of the composite, (b) the coordinates of a fiber in the system.

$$\epsilon_c = \epsilon_f = \epsilon_m \tag{1}$$

Since the total load F_c is shared between the fiber and matrix, we have

$$F_c = F_f + F_m \tag{2}$$

or in terms of stresses

$$\sigma_c A(c) = \sigma_f A(f) + \sigma_m A(m) \tag{3}$$

where $A(c)$, $A(f)$, and $A(m)$ are the corresponding cross section areas of the composite, the fiber and the matrix. Furthermore

$$\sigma_c = \sigma_f \frac{A(f)}{A(c)} + \sigma_c \frac{A(m)}{A(c)} = \sigma_f A_f + \sigma_c A_m \tag{4}$$

At the cross section in question, the fiber area fraction is equal to the volume fraction, i.e.,

$$A_f = \frac{A(f)}{A(c)} = V_f \tag{5}$$

so that

$$\sigma_c = \sigma_f A_f + \sigma_c A_m = \sigma_f V_f + \sigma_c V_m \tag{6}$$

The elastic modulus of the composite in this direction can then be readily obtained from the above equation as

$$E_c = E_f V_f + E_m V_m \tag{7}$$

Equations (6) and (7) are the well-known Rule of Mixtures for composite stress and tensile modulus, respectively.

It is now clear that the critical condition for the applicability of the Rule of Mixtures in this case is Equation (5), i.e., the relationship between A_f and V_f. At such cross sections of a unidirectional composite, the fiber volume and area fractions are identical. However, this condition will no longer occur if fibers, instead of being parallel, are oriented in different directions. In that case, the composite is no longer a unidirectional one. When we cut arbitrarily cross sections in different directions of the composite, the number of fiber ends exposed on the cross sections will no longer be identical, and the shapes of the fiber ends on one composite cross section will not all be circular due to various orientations of the fibers. Numerically, for such composites, the fiber area fraction at a cross section will in general be different from the overall system fiber volume fraction V_f.. Moreover, although the system fiber volume fraction V_f remains a constant, the fiber area fraction A_f will change from direction to direction in the composite, unless fibers are oriented in such a way that the composite becomes an isotropic one.

Apparently, the key issue now is to try to establish a general relationship between the system fiber volume fraction V_f and the fiber area fraction A_f, for once such a relationship is available in place of Equation (5), we can still apply the Rule of Mixtures to calculate the elastic properties of other composites than just unidirectional ones.

In fact, such a general relationship has recently been developed by the present author in /33/. In the next section, we will first very briefly introduce the related theoretical background and this V_f–A_f relationship. We will then compare the new approach with the existing theories as well as the previously published experimental data in a later section.

2.2. A general V_f–A_f relationship

In the spatial curvilinear coordinate system in Figure 1(b), the orientation of a fiber in the composite can be defined uniquely by a pair of angles (θ, ϕ), provided that the polar angle $0 \leq \theta \leq \pi$ and the base angle $0 \leq \phi \leq \pi$. Meanwhile, the direction of any cross section in the composite can be represented by the direction of its normal defined by another angle pair (Θ, Φ). Because of the existing anisotropy, the fiber area fraction A_f (Θ, Φ) on a cross section will be a function of the direction of the cross section (Θ, Φ), although on this given cross section (Θ, Φ) we still have the relation between the fiber and matrix area fractions

$$A_f(\Theta,\Phi) + A_m(\Theta,\Phi) = 1 \tag{8}$$

Next, we define a probability density function (*pdf*) $\Omega(\theta, \phi)$ to specify the fiber orientation, subject to the normalization condition

$$\int_0^\pi d\theta \int_0^\pi d\phi \Omega(\theta,\phi) \sin\theta = 1 \tag{9}$$

Consider an arbitrary cross section of direction (Θ, Φ) in the composite. Pan has proved in /33/ that the fiber area fraction $A_f(\Theta, \Phi)$ is related to the system overall fiber volume fraction V_f through the probability density function as

$$A_f(\Theta,\Phi) = \Omega(\Theta,\Phi)V_f \tag{10}$$

where Ω (Θ, Φ) is the value of the *pdf* in direction (Θ, Φ). This equation reveals that generally the fiber area fraction is a function of direction, and is hence different from the constant system fiber volume fraction. The difference is caused by fiber misorientation, if the matrix is considered isotropic. The only case where $A_f = V_f$ is when the density function $\Omega(\Theta, \Phi) = 1$; this happens only in the composites made of fibers unidirectionally oriented in direction (Θ, Φ).

Replacing Equation (5) with the above $V_f - A_f$ relationship in Equation (10), we obtain the expression of the elastic tensile modulus in direction (Θ, Φ) for a composite with arbitrary fiber orientations

$$E_c(\Theta,\Phi) = E_f\Omega(\Theta,\Phi)V_f + E_m(1-\Omega(\Theta,\Phi)V_f) \tag{11}$$

Consequently, we can now predict the tensile modulus, as well as its direction dependence, or the anisotropy, of the composite once the *pdf* is given.

3. PREDICTIONS AND DISCUSSIONS

Although progress has been made continuously, there is still no simple technique, experimental or analytical, for the acquisition of the *pdf* function for a given composite. We hence focus here on a special case: random fiber orientation so that $\Omega(\Theta, \Phi)$ will be a constant independent of directions. The composites therefore become isotropic.

3.1. For 2-D cases

We start by looking at a planar 2-D case. In fact, 2-D random fiber orientation is of practical importance, as mentioned in /21/. In the case of injection moulded objects, fiber orientation distribution is dependent only on the base angle if the direction of flow is along the composite principal axis. In sheet moulding compounds it is reasonable to assume that the short fibers all lie within a plane and the problem is again reduced to a two-dimensional one. In either case, the fiber orientation can be considered independent of the polar angle; we can hence set in the following analysis $\theta = \Theta = \frac{\pi}{2}$.

The *pdf* function thus becomes

$$\Omega(\phi) = \Omega_0 \tag{12}$$

where Ω_0 is a constant whose value is determined using the normalization condition as

$$\Omega_0 = \frac{1}{\pi} \tag{13}$$

Then the tensile modulus in Equation 11 is

$$E_c^{2D} = E_f \frac{V_f}{\pi} + E_m(1 - \frac{V_f}{\pi}) \tag{14}$$

According to several sources, for instance in /20,29/ and as validated experimentally in /30/, the Rule of Mixtures is also applicable to the Poisson's ratio of a unidirectional composite, then by the same reasoning as in the case of tensile modulus, we can derive for the Poisson's ratio

$$\nu_c^{2D} = \nu_f \frac{V_f}{\pi} + \nu_m(1 - \frac{V_f}{\pi}) \tag{15}$$

where ν_f and ν_m are the Poisson's ratios for the fiber and matrix, respectively.

3.2. For 3-D cases

The 3-D random fiber composite is rarely used in practice, mainly because of the difficulty in controlling fiber orientation in three dimensions. In this case, the fiber orientation will be independent of both the polar and base angles. Therefore, the normalization condition gives

$$\Omega(\Theta, \Phi) = \frac{1}{2\pi} \tag{16}$$

The tensile modulus becomes

$$E_c^{3D} = E_f \frac{V_f}{2\pi} + E_m(1 - \frac{V_f}{2\pi}) \tag{17}$$

and the Poisson's ratio

$$\nu_c^{3D} = \nu_f \frac{V_f}{2\pi} + \nu_m(1 - \frac{V_f}{2\pi}) \tag{18}$$

For both 2-D and 3-D random fiber composites, since we are dealing with isotropic systems, we have the intrinsic relation between the tensile, shear moduli and the Poisson's ratio of the composites as

$$G_c = \frac{E_c}{2(1 + \nu_c)} \tag{19}$$

So the shear modulus of the system can be calculated as well.

In order to compare our new model with other typical theories as well as with experimental results, we first summarize several theoretical models in Table I. The widely used Halpin-Tsai equations /20/ are included in the table where the calculation procedures for the two moduli E_L and E_T can be found /20/. The model by Christensen /26/ was developed using a combined laminate analogy and fiber orientation function techniques to derive the results. Since the complete expressions for these elastic properties are quite long, Table I shows the versions simplified by Christensen himself. The equations by Manera /9/ are also approximations derived using the laminate analogy approach. As a reference, results by Cox /31/ are listed as well. The equations corresponding to our new model are also summed up in Table I.

Table II from /35/ and III from /9/ provide some of the published experimental data for random fiber composites. Conversions have been done by the present author to change the original data into the preferred units. The fiber aspect ratio s in Table III is the mean value calculated using the fiber size provided in /9/. It has to be pointed out that since a completely random fiber orientation is difficult to achieve, and partial alignment of fibers in composite samples is almost inevitable, especially when the system fiber volume fraction V_f is large, the accuracy of the experimental data is often only relative.

Figure 2(a), (b) and (c) compare the theoretical predictions by the various models with the experimental data in Tables II and III. In Figure 2(a), the agreement between the experimental data and the prediction by the new model has been excellent, better than those by either Halpin-Tsai or Christensen and Manera. In Figure 2(b), however, the new model predicts results that coincide with the data at a low V_f level, but not as well as the prediction by Christensen when V_f is greater. In Figure 2(c), although the predictions by other theories are closer to the experimental data, the

Table I
Various theoretical models

Item	Christensen	Manera	Pan
$E_c^{2D} =$	$\frac{V_f}{3}E_f + (1+V_f)E_m$	$(\frac{16}{45}E_f + 2E_m)V_f + \frac{8}{9}E_m$	$\frac{V_f}{\pi}E_f + (1-\frac{V_f}{\pi})E_m$
$E_c^{3D} =$	$\frac{V_f}{6}E_f + [(1+(1+V_f)]E_m$	$(\frac{2}{15}E_f + \frac{3}{4}E_m)V_f + \frac{1}{3}E_m$	$\frac{V_f}{2\pi}E_f + (1-\frac{V_f}{2\pi})E_m$
$G_c^{2D} =$		$(\frac{2}{15}E_f + \frac{3}{4}E_m)V_f + \frac{1}{3}E_m$	$\frac{E_c^{2D}}{2(1+\nu_c^{2D})}$
$G_c^{3D} =$			$\frac{E_c^{3D}}{2(1+\nu_c^{3D})}$
$\nu_c^{2D} =$		$\frac{1}{3}$	$\frac{V_f}{\pi}\nu_f + (1-\frac{V_f}{\pi})\nu_m$
$\nu_c^{3D} =$			$\frac{V_f}{2\pi}\nu_f + (1-\frac{V_f}{2\pi})\nu_m$
restrictions	$V_f < 0.2$	$0.1 \le V_f \le 0.4$ 2 GPa $\le E_m \le$ 4 GPa $\nu_m \sim 0.4$	
Item	Halpin-Tsai		Cox
$E_c^{2D} =$	$\frac{3}{8}E_L + \frac{5}{8}E_T$		$\frac{V_f}{3}E_f$
$E_c^{3D} =$			$\frac{V_f}{6}E_f$
$G_c^{2D} =$	$\frac{1}{8}E_L + \frac{1}{4}E_T$		$\frac{V_f}{8}E_f$
$G_c^{3D} =$			$\frac{V_f}{15}E_f$
$\nu_c^{2D} =$	$\frac{E_c^{2D}}{2G_c^{2D}} - 1$		$\frac{1}{3}$
$\nu_c^{3D} =$			$\frac{1}{4}$

Table II
Experimental results for 2-D case by Lee /35/

glass fiber, $E_f = 72.40GPa$, $s = \frac{l_f}{d_f} = 675$				
polystyrene, $E_m = 3.24GPa$				
V_f	0.044	0.095	0.152	0.218
$E_c(GPa)$	4.27	5.45	6.48	8.07
poly(styrene-co-acrylonitrile), $E_m = 3.65GPa$				
V_f	0.044	0.095	0.152	0.218
$E_c(GPa)$	4.55	5.52	7.72	9.58

Table III
Experimental results for 2-D case by Manera /9/

fiber, $E_f = 73.0GPa$, $\nu_f = 0.25$, $s = 6600$					
matrix, $E_m = 2.25GPa$, $\nu_m = 0.4$					
V_f	$E_c(GPa)$	V_f	$G_c(GPa)$	V_f	ν_c
0.171	6.872	0.085	2.551	0.181	0.347
0.174	7.221	0.094	1.595	0.183	0.333
0.232	8.184	0.119	1.935	0.244	0.337
0.236	8.142	0.129	2.292	0.247	0.345
0.265	9.749	0.139	2.649	0.281	0.325
0.269	10.168	0.203	2.741	0.287	0.327
0.330	11.662			0.348	0.329
0.334	12.123			0.351	0.319
0.358	13.114			0.375	0.331
0.360	11.941			0.380	0.324

results from the present model are still within the error range specified by Manera in /9/.

It may be useful to mention that the data in Table III covers the results over a higher V_f level than those in Table II, and at a higher V_f level, a complete randomness of fiber orientation is more difficult to achieve. Furthermore, Christensen, trying to find reliable experimental data to verify his theory, commented in /26/ that "One set of data for which it appears that care had been exercised in obtaining a random (two-dimensional) orientation of the fibers is that of Lee." Therefore, it is justifiable to consider the data in Table II to be more accurate.

Nevertheless, from these comparisons, one conclusion we can draw is that the predictions by the new model agree at least as well with the experimental data as those by the other theories compared.

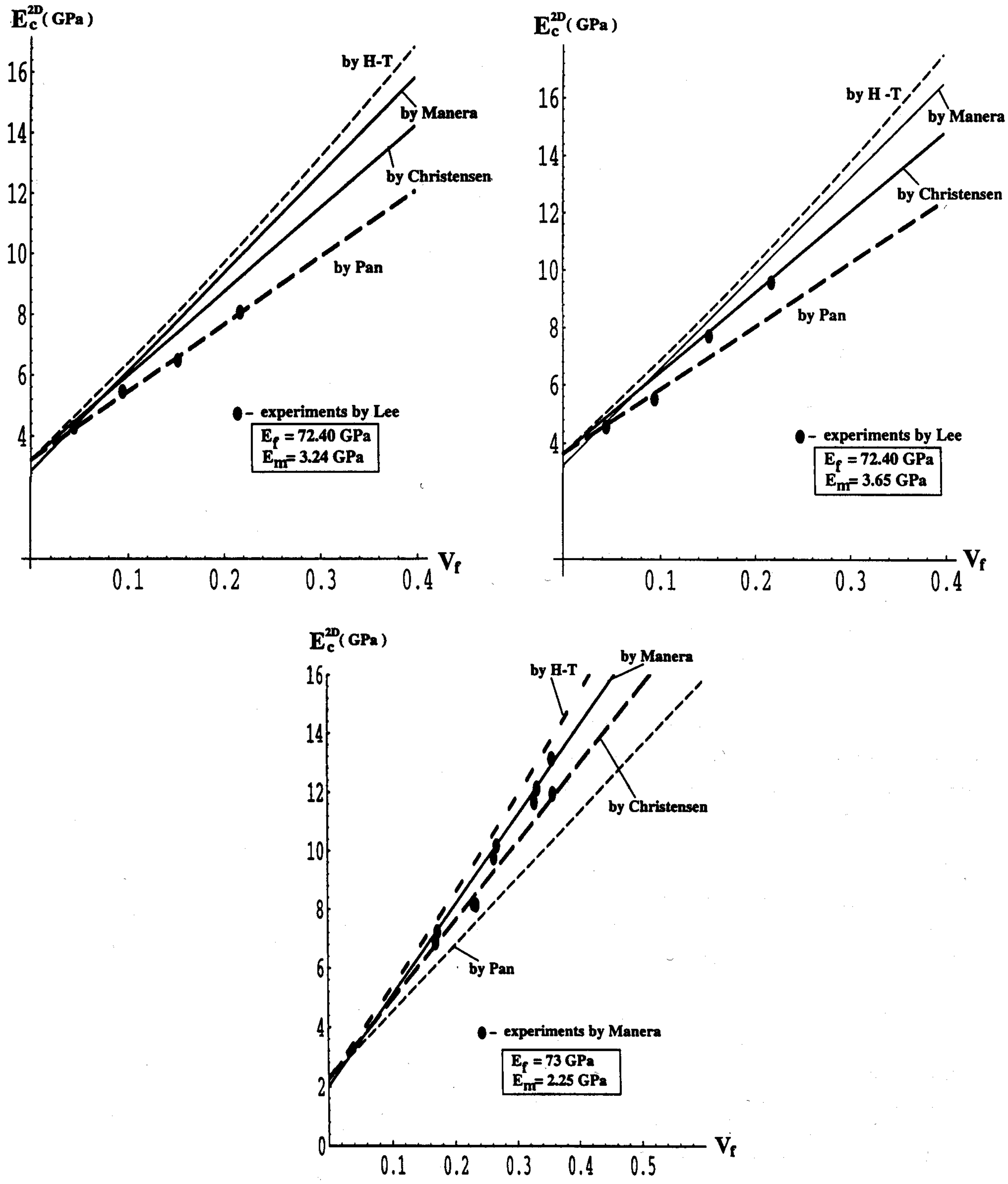


Fig. 2: Relationship between tensile modulus E_c^{2D} and fiber volume fraction V_f:
(a) comparisons between theories and experimental result by Lee /35/,
(b) comparisons between theories and experimental result by Lee /35/,
(c) comparisons between theories and experimental result by Manera /9/.

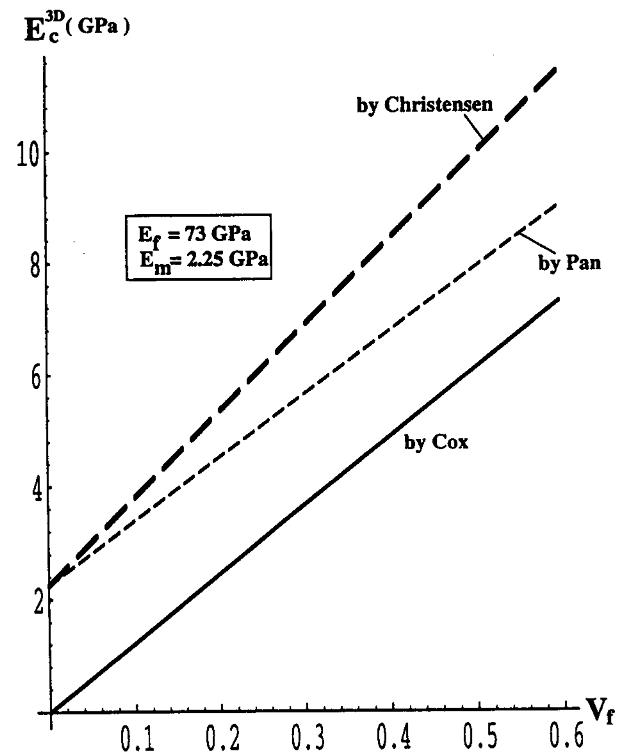
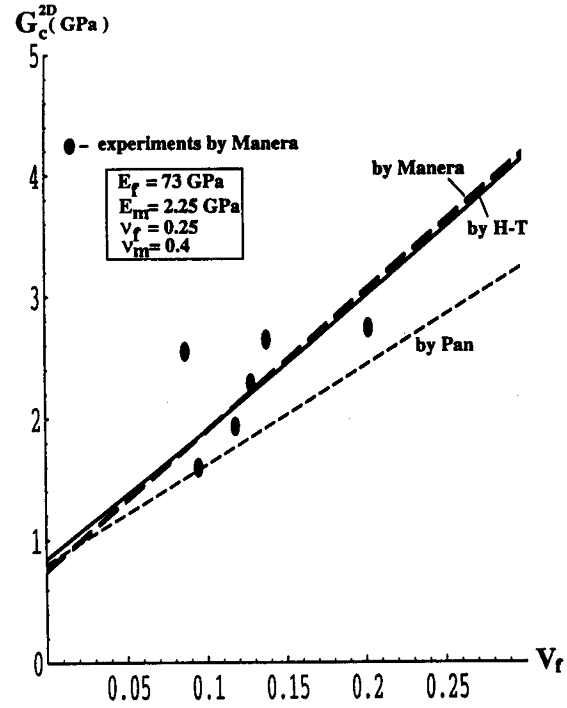


Fig. 3: Theoretical predictions of relationship between E_c^{3D} and V_f.

As to the 3-D case, since there are no reported experimental results for the aforementioned reasons, we compare only the predictions in Figure 3 by the theoretical models. It is seen that Christensen's model yields the highest value, and Cox's model the lowest one, due mainly to its excluding the contribution from the matrix, and thus only serves as a reference for comparison. The results from the new model lie in between the two. It is also learnt from the results in Figures 2 and 3 that for the same fiber volume fraction level, the tensile modulus in the 3-D case is smaller than that of a 2-D composite, when the fiber used is stiffer than the matrix.

The comparison between the models and the experimental data in terms of the shear modulus for both 2-D and 3-D cases is illustrated in Figure 4. The three predictions for the 2-D case in Figure 4(a) seems to be all in agreement with the experimental data.

For the 3-D case, as only two models, those by Cox and Pan in Table I, are applicable here, their predictions are shown in Figure 4(b). Again, for the



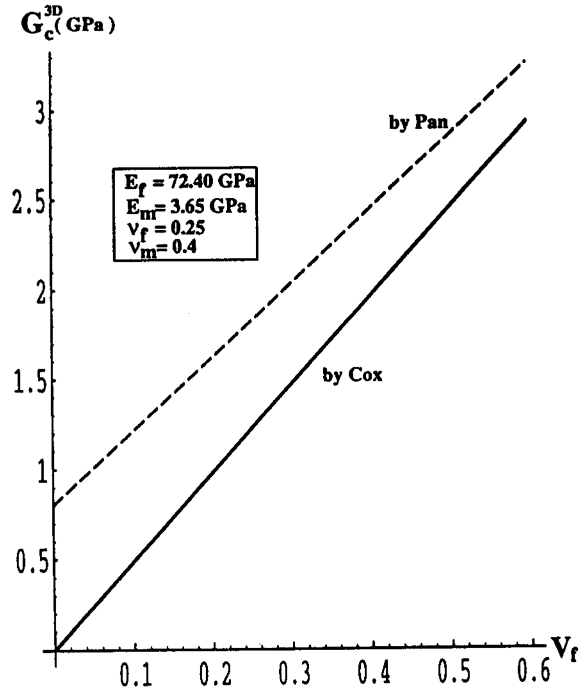


Fig. 4: Relationship between shear moduli and fiber volume fracture V_f:
(a) comparisons of G_c^{2D} between theories and experimental result by Manera /9/,
(b) comparisons of G_c^{3D} between theories.

same fiber and matrix at the same V_f level, G_c^{3D} is lower than G_c^{2D}. Here Cox's result is also lower for the reason mentioned above.

Figure 5 depicts the results for the Poisson's ratios for both 2-D and 3-D composites, where the predictions by Manera and Cox are all constants. In the 2-D case, the results from Manera and Cox provide the lower bound and that from H-T prediction gives the upper bound, whereas the prediction from the present model is in the middle. Although Cox and Manera's models yield results closer to the experimental data, this should not be taken as evidence to disprove other models, because Cox and Manera's models are such rough approximations that they exclude both fiber and matrix properties as well as their volume fractions.

In the 3-D case, Figure 5(b) illustrates the predictions by Cox's and the present models. Although we have no data to validate our prediction, we are certain that the result by Cox cannot be correct, again because all material and system parameters are ignored.

4. CONCLUSION

The value of the system fiber volume fraction V_f in a composite is a constant, and is in general different from the fiber area fraction A_f in a given direction of the composite, which in most cases is not a constant but a function of fiber orientation in the composite.

By establishing the relationship between V_f and A_f., combined with the Rule of Mixtures, we can predict, for both 2-D and 3-D cases, the tensile modulus and the Poisson's ratio of a random fiber composite and, consequently, calculate its shear modulus by means of the constitutive restraints on the three parameters.

For the same fiber volume fraction, the values of both tensile and shear moduli are found to be higher for a planar composite than for a 3-D case, both with random fiber orientation.

This new model has been proved as good as other existing models, but simpler with no limit, unlike some models, on the value of the fiber volume fraction V_f. Moreover, the present theory can be applied to other cases besides random fiber orientation, as long as the fiber orientation density function can be obtained.

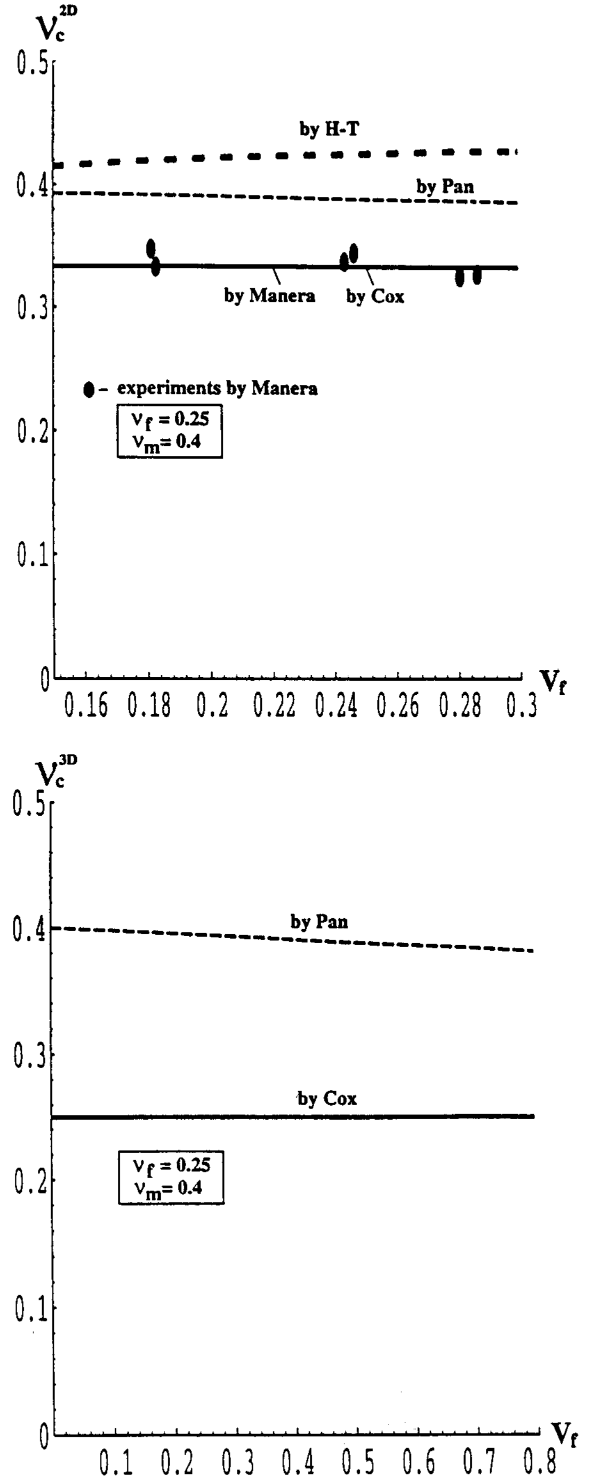


Fig. 5: Relationship between Poisson's ratios and fiber volume fraction V_f.
(a) comparisons of ν_c^{2D} between theories and experimental result by Manera /9/,
(b) comparisons of ν_c^{3D} between theories.

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