Structural Anisotropy, Failure Criterion, and Shear Strength of Woven Fabrics

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ABSTRACT

For structural and load-carrying applications, more detailed information about woven fabrics, which are increasingly used as reinforcements for composites, is desirable for design purposes. This paper investigates the issues related to woven fabric strength. First, the direction dependence or anisotropy of tensile strength as well as the breaking strain and initial modulus of the fabrics are examined experimentally. A harmonic expression is then adopted to approximate the experimental results so that this tensile strength anisotropy can be expressed analytically. Moreover, the Tsai-Wu failure criterion is used, assuming it is valid for woven fabrics, at least at the first quadrant where failure stresses are all tensile. Unknown coefficients in the failure criterion are determined based on the experimental results. Fabric shear strength is then predicted based on the measured uniaxial tensile strengths of the fabric at the principal and off-axial directions. Influences of various directions of the off-axial tensile test on predictions of fabric shear strength are also studied.

Orthogonal or biaxial woven fabrics exhibit good dimensional stability in the warp and weft directions and offer the highest yarn packing density in relation to fabric thickness. Also, woven fabrics provide more balanced properties in the fabric plane than unidirectional laminae; the bi-directional reinforcement in a single layer of a fabric gives rise to enhanced impact resistance. All these advantages along with ease of handling and low fabrication cost as well as light weight have made these fabrics attractive for structural applications and reinforced composites. But for these applications, a better understanding of the mechanical behavior of woven fabrics is indispensable.

In a previous paper [14], we dealt with the *in-situ* behavior of and interactions between yarns in woven fabrics under tension and their effects on the ultimate tensile strength of the fabrics. Much of our effort focused on the effects of the statistical variations of yarn properties on fabric tensile strength. By incorporating these factors into the analysis, we were able to predict fabric tensile strength in the principal or warp and filling directions under uniaxial and biaxial extension.

Moreover, woven fabrics are well known for their property-direction dependence or property anisotropy. An experienced tailor understands this well enough to choose different "grains" for different pieces of a garment, so that properly draped, elegant apparel can be made. For structural applications, however, this anisotropy will present a problem of irregularities in terms of performance or load-carrying capacity. One objective in our work is to evaluate and predict this property anisotropy.

Woven fabrics are not only highly anisotropic, but also dimensionally changeable, very susceptible to external loading and to its historical conditions. The important fabric properties critical to structural applications include tensile strength, in-plane shear strength, and normal compressive (in the thickness direction) strength, as well as in-plane compressive or buckling strength.

Predicting fabric strength has significance both theoretically and practically, because except for uniaxial tensile strength, experimental determination of all other strengths is tedious, with no convenient test methods and instruments available, and very costly. Sheet-form materials, whose flexural and torsional rigidity are very low, particularly require very elaborate devices to measure shear and buckling strengths.

Kilby [7] is probably the first researcher to deal with the mechanistic anisotropy of a woven fabric. He derived the so-called generalized modulus of a fabric, expressing the fabric tensile modulus in relation to the test direction. In this study, however, we will focus on the anisotropy of fabric strength. Following earlier work [14], in which we proposed a more realistic approach to predict fabric tensile strengths at the principal directions under uniaxial and biaxial extension, we present in this study an attempt to investigate the direction

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dependence or anisotropy of the tensile strength of a woven fabric using a technique described by Cheng and Tan [2] based on experimental results.

Furthermore, by using the Hill-type failure criterion [5, 21, 22, 23] widely applied in studying fiber reinforced composites [1, 6, 15], wood materials [8, 11], and paper and geotextiles products [9, 10, 16, 17, 18, 19], we attempt to predict the shear strength of woven fabrics using measured tensile strengths.

Predicting Tensile Strength Anisotropy

One approach of strength prediction is to use the so-called failure criterion to derive other strength terms based on given values of strengths tested in a few particular directions. Of the various failure criteria for anisotropic materials, only three [23] have received wide attention, those of Hill [5], Hoffman [6], and Tsai-Wu [22]. Both Hill's and Hoffman's theories are limited to orthotropic materials with plastic incompressibility. In this respect, the Tsai-Wu theory has wider applicability. The basic assumption for the Tsai-Wu theory is that there exists a failure surface in the stress space, which can be expressed in terms of a stress tensor polynomial function. In general, however, to apply the function, one has to know the compressive and shear strengths of the material in addition to its tensile strengths.

Cheng and Tan have proposed an alternative technique [2], using a harmonic cosine series to represent the off-axial tensile strength of an anisotropic polymeric sheet or plate at any direction. That is,

$$X_\phi = (\Sigma C_n \cos n_\phi)^{-1} \quad , \qquad (1)$$

where $n = 0, 2, 4, \ldots$, and X_ϕ is the tensile strength at direction ϕ as defined in Figure 1. C_n are the factors to be determined using given or experimentally obtained tensile strength values in a few particular directions where tests are easy to do. Higher prediction accuracy using Equation 1 can be achieved with more pre-tested strength values, so that more C_n factors can be derived.

To demonstrate Equation 1, we provide a practical example. The same five fabrics in reference 14 are used here; their properties are listed in Table I. The uniaxial tensile strengths of fabric strips cut at seven different directions ϕ, as illustrated in Figure 1, where L indicates the warp and T the filling directions, are measured according to the ASTM D1682-65 cut strip method at a cross-head speed of 200 mm/min, and the testing re-

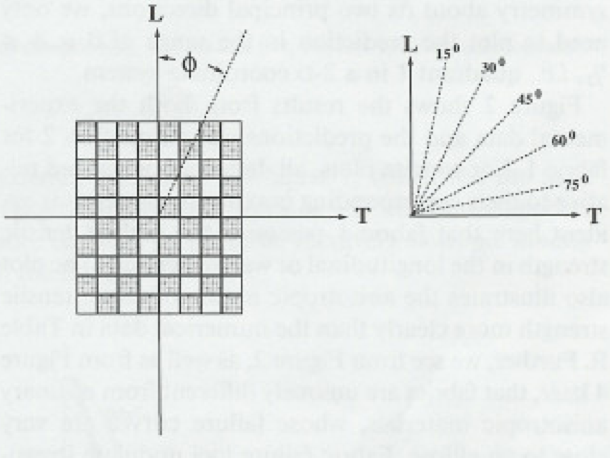

FIGURE 1. Coordinate system (a) and testing directions (b) on a fabric.

TABLE I. Data of the fabric samples.[a]

	Fabric 1, 100% PET-F plain	Fabric 2, 100% acetate-F plain	Fabric 3, 100% acetate-F satin	Fabric 4, 100% cotton-S plain	Fabric 5, 100% cotton-S plain
Fabric cover factor, $\left(\sqrt{\text{tex}}\,\frac{\text{yarn}}{\text{mm}}\right)$					
Warp	7.14	7.46	8.81	16.87	16.28
Weft	8.74	12.55	15.02	14.21	14.67
Yarn strength, N/tex					
Warp	1.155	0.365	0.386	0.098	0.099
Weft	0.764	0.091	0.093	0.073	0.079
Fabric strength, (N/thread)					
Warp	4.40	2.09	1.84	3.06	3.14
Weft	3.78	3.87	4.04	2.84	3.64
Fabric strength, (N/tex)					
Warp	1.341	0.537	0.461	0.077	0.085
Weft	0.768	0.120	0.129	0.069	0.083

[a] F is the filament yarn and S the staple yarn.

sults are provided in Table II. As we have seven different directions, Equation 1 can be expanded into

$$X_\phi = (C_0 + C_2 \cos 2\phi + C_4 \cos 4\phi + C_6 \cos 6\phi + C_8 \cos 8\phi + C_{10} \cos 10\phi + C_{12} \cos 12\phi)^{-1} , \quad (2)$$

with seven unknown C_n factors.

By substituting the tested strength values at direction ϕ in Table II into Equation 2, we can form seven simultaneous equations from which the seven unknown C_n factors can be determined. Equation 2 can then be used to predict tensile strength at any direction ϕ besides the seven directions already tested.

The C_n values for the five different fabrics are thus calculated and also provided in Table II. Since a fabric can be treated as an orthotropic material with property symmetry about its two principal directions, we only need to plot the prediction in the range of $0 \le \phi \le \pi/2$, *i.e.*, quadrant 1 in a 2-D coordinate system.

Figure 2 shows the results from both the experimental data and the predictions using Equation 2 for fabric 1. For concise plots, all data are normalized relative to their corresponding maximum values. It is evident here that fabric 1 possesses the highest tensile strength in the longitudinal or warp direction. The plot also illustrates the anisotropic nature of fabric tensile strength more clearly than the numerical data in Table II. Further, we see from Figure 2, as well as from Figure 4 later, that fabrics are uniquely different from ordinary anisotropic materials, whose failure curves are very close to an ellipse. Fabric failure loci undulate irregularly, due certainly to the fact that the structure consists of discrete yarns interlaced at two orthogonal directions. Possible yarn-yarn relative movements and interactions at the crossing points are likely responsible for this irregular failure curve. Note that for such an irregular shape, it is perhaps more advantageous to approximate the curve using a harmonic expression, as in Equation 2, than a polynomial function of a regular failure criterion.

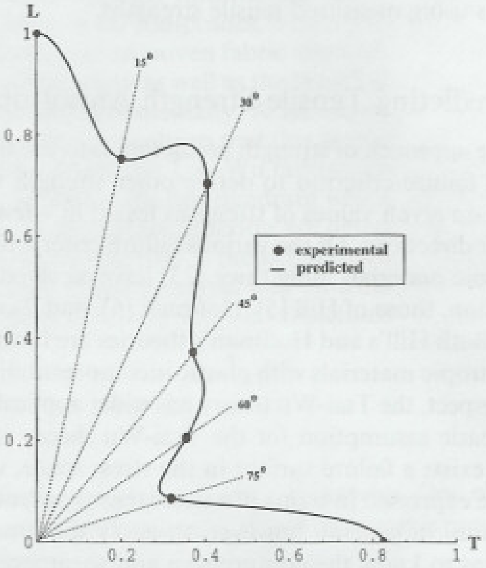

FIGURE 2. Fabric tensile strength anisotropy determined experimentally and analytically.

To understand the contribution of each harmonic component in Equation 2 toward overall system anisotropy, we have constructed Figures 3a and b for fabric 1, where the numerical number n indicates that the

TABLE II. Tested fabric strength versus direction.

	Breaking load, 9.8 · N				
Degrees	Fabric 1	Fabric 2	Fabric 3	Fabric 4	Fabric 5
0°, L	43.95 (0.60)[a]	19.45 (0.22)	26.70 (0.38)	20.59 (1.26)	21.17 (0.95)
15°	34.27 (0.87)	11.03 (0.62)	15.84 (0.33)	10.19 (1.12)	9.97 (1.32)
30°	35.73 (1.78)	12.87 (1.06)	19.61 (1.68)	14.01 (0.36)	15.03 (4.00)
45°	23.08 (1.46)	15.10 (0.49)	21.08 (0.26)	16.32 (1.27)	17.74 (2.42)
60°	18.01 (0.92)	13.88 (0.63)	15.47 (0.60)	12.16 (0.90)	12.58 (0.59)
75°	14.51 (1.35)	11.88 (0.42)	13.12 (0.79)	8.57 (0.87)	8.06 (1.46)
90°, T	36.30 (1.14)	20.14 (0.70)	27.63 (0.32)	16.25 (0.60)	20.07 (0.57)
C_n					
C_0	1.83	1.48	1.56	1.66	1.68
C_2	0.74	−0.06	0.16	0.19	0.20
C_4	−0.16	−0.02	0.01	0.17	0.22
C_6	−0.37	0.03	−0.13	−0.03	−0.08
C_8	−0.33	−0.31	−0.40	−0.47	−0.57
C_{10}	−0.27	0.01	−0.05	−0.03	−0.09
C_{12}	−0.24	−0.14	−0.16	−0.23	−0.30

[a] Data in the parenthesis are standard deviation values.

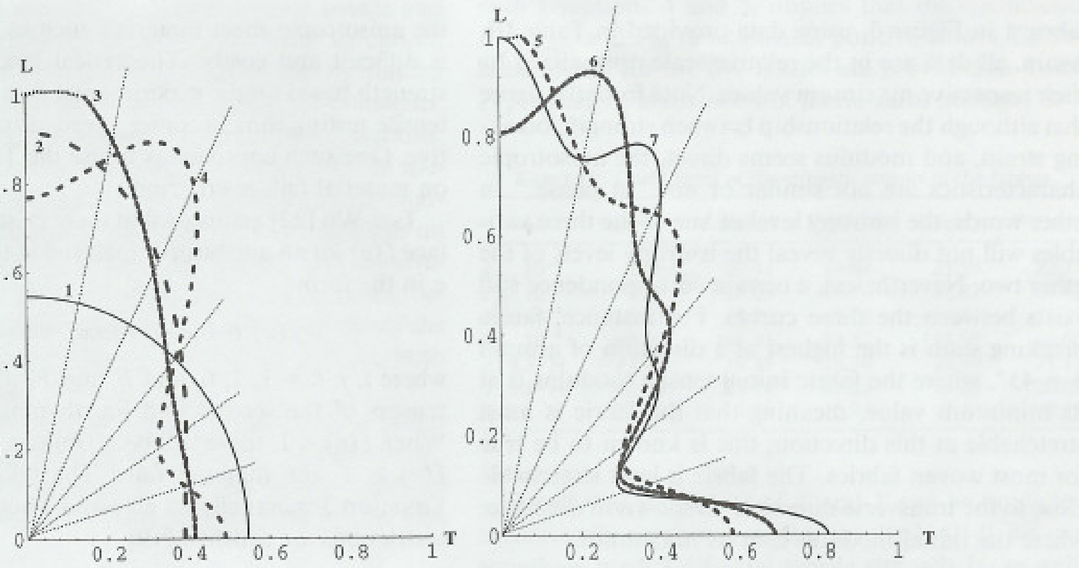

FIGURE 3. Contributions of various harmonic components: (a) first four components, and (b) first five, six, and all seven components.

first n components in the right hand side of Equation 2 are used in constructing the curve. We can conclude that because the first constant coefficient C_0 describes the isotropic element of fabric mechanical behavior, so the corresponding curve in Figure 3a is a perfect circle representing the direction-independent element of strength. Adding in the harmonic components 2 and 3 makes the curves more elliptical, with the longer axis in the longitudinal direction. Tensile strength is highest in the longitudinal direction, and then decreases monotonically as the bias direction ϕ increases to 90°, coinciding with the filling or transverse direction. The harmonic components 4, 5, and 6 change the ellipses into more irregular curves, reflecting the instabilities of the fabric structure. Finally, with contributions from all seven components, a complete strength-direction dependent behavior of the material is depicted. Since most ordinary anisotropic materials yield an elliptical failure surface, it is intuitive to conclude that components 4, 5, 6, and 7 represent intrinsic properties unique to woven fabrics.

Figure 4 compares all five fabrics in terms of their tensile strength anisotropy. Fabrics 1, 4, and 5 are strongest in the warp direction, whereas fabrics 2 and 3 have maximum tensile strength in the transverse or filling direction. No maximum tensile strength is achieved in the bias directions for any of the fabrics. If we take the closeness of a curve to a circle as an indicator of the strength isotropy of the material, a curve identical to a circle would be an ideally isotropic material. The curve for fabric 1, which has the least strength drop in direction $\phi = 15°$ and hence appears more circular, seems to be relatively isotropic among all the fabrics.

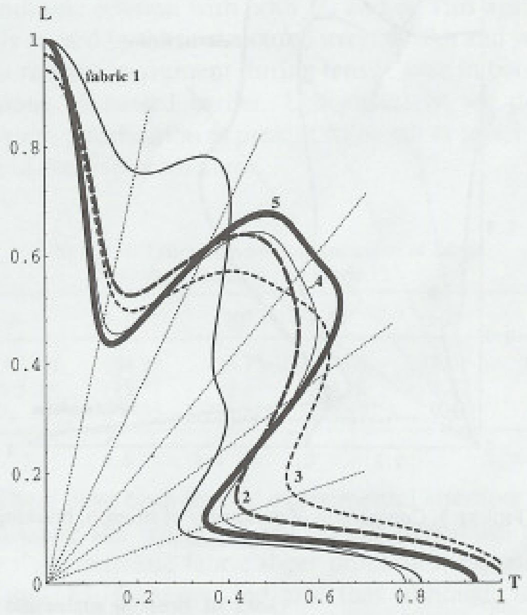

FIGURE 4. Comparison of five different fabrics.

Additionally, we compare the anisotropy curves of the strength, breaking strain, and initial modulus for fabric 1 in Figure 5, using data provided in Table III. Again, all data are in the relative scale normalized by their respective maximum values. Note from the figure that although the relationship between strength, breaking strain, and modulus seems direct, the anisotropic characteristics are not similar or not "in phase." In other words, the isotropy level of one of the three variables will not directly reveal the isotropy levels of the other two. Nevertheless, a certain correspondence still exists between the three curves. For instance, fabric breaking stain is the highest at a direction of around $\phi = 45°$, where the fabric initial tensile modulus is at its minimum value, meaning that the fabric is most stretchable at this direction; this is known to be true for most woven fabrics. The fabric is least stretchable close to the transverse direction, as shown in the figure where the initial modulus is at its maximum.

Predicting Shearing Strength Based on Uniaxial Tensile Strengths

As we mentioned in the introduction, instrumental measurement with high accuracy of shear strength for the anisotropic sheet materials such as woven fabrics is difficult and costly. Theoretical prediction of this strength based on the experimental data from uniaxial tensile testing thus becomes a very attractive alternative. One such approach is to use the Tsai-Wu theory on material failure criterion.

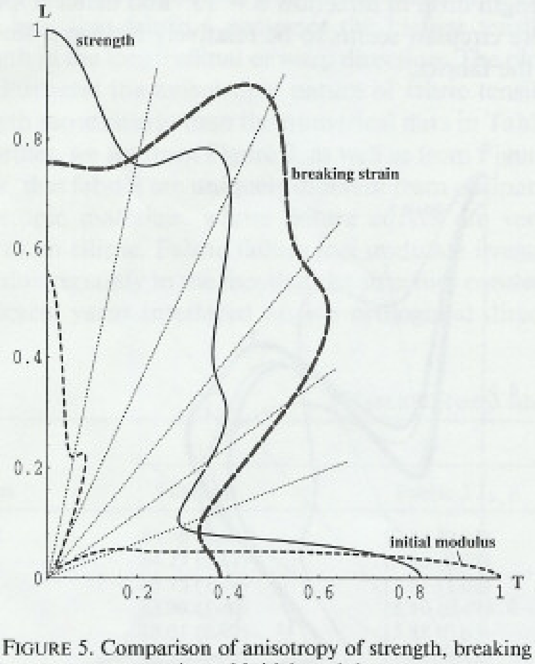

FIGURE 5. Comparison of anisotropy of strength, breaking strain and initial modulus.

Tsai-Wu [22] assumed that there exists a failure surface $f(\sigma)$ for an anisotropic material in the stress-space σ in the form

$$f(\sigma) = F_i\sigma_i + F_{ij}\sigma_i\sigma_j = 1 \quad , \tag{3}$$

where $i, j, k = 1, 2, 6$, and F_i and F_{ij} are the strength tensors of the second and fourth rank, respectively. When $f(\sigma) < 1$, there will be no failure, whereas when $f(\sigma) \geq 1$, the material fails. The failure surface of Equation 3 is actually an ellipsoid when the following restrictions are satisfied [19]:

$$F_{11}F_{66} > 0 \quad , \tag{4}$$

$$F_{22}F_{66} > 0 \quad , \tag{5}$$

and

$$F_{11}F_{22} - F_{12}^2 \geq 0 \quad . \tag{6}$$

For fixed values of shear stress σ_6, the equation scribes ellipses in the σ_1-σ_2 plane. For orthotropic sheet materials, the analysis is restricted to a plane stress state, so Equation 3 can then be reduced to [17]

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + 2F_{12}\sigma_1\sigma_2 + F_{66}\sigma_6^2 = 1 \quad . \tag{7}$$

Let us assume that such a failure criterion is also valid for materials like woven fabrics; we can thus use this failure criterion to estimate fabric shear strength. If we choose the coordinate system in Figure 1, we will have

$$\sigma_1 = \sigma_L, \quad \sigma_2 = \sigma_T, \quad \sigma_6 = \tau_{LT} = \tau_{TL} \quad .$$

To determine the four coefficients F_{ij} according to Tsai and Wu [22], we need to know the uniaxial tensile and compressive strengths in the L and T directions, and the pure shear in-plane strength as well as the uniaxial tensile strength of the material in a bias direction.

As Rowlands *et al.* [17] pointed out, however, Equation 3 implies equal uniaxial strengths in tension and compression for the material concerned. This limitation is the result of an assumption associated with the theory that hydrostatic stress has no effect on material

TABLE III. Breaking strains and initial moduli versus directions (for fabric 1).

	0°, L	15°	30°	45°	60°	75°	90°, T
Breaking strain, %	49.11	51.97	64.90	52.28	39.53	22.59	24.82
	(3.41)	(3.16)	(2.97)	(3.10)	(1.26)	(1.67)	(1.26)
Initial moduli, GPa	1.59	0.65	0.25	0.08	0.17	0.57	2.88
	(0.60)	(0.18)	(0.43)	(0.28)	(0.19)	(0.21)	(1.29)

strength. Since theories that predict equal tensile and compressive strengths are restricted only to certain materials, Norris [11] avoided this problem by making F_{11} and F_{22} functions of stress, instead of constants. He then divided the stress plane into four quadrants, so that the unknown coefficients could be derived specifically for each quadrant with different stress characteristics. In our particular case, in order to focus on predicting fabric shear strength without the involvement of fabric in-plane compressive or buckling behavior, we will only consider the first quadrant of the four, where the stresses $\sigma_L \geq 0$ and $\sigma_T \geq 0$ are both tensile in nature.

Denoting the uniaxial tensile strengths of the fabric in both L and T directions as X and Y, respectively, since the fabric strengths are on the strength surface defined in Equation 7, this implies that in the first quadrant,

$$F_{11} = \frac{1}{X^2} \tag{8}$$

and

$$F_{22} = \frac{1}{Y^2} \quad . \tag{9}$$

The various Hill-type predictions differ from one another only by the manner in which the coefficient F_{12} is determined [17]. Here, we choose Norris' result [11]:

$$F_{12} = -\frac{1}{2XY} \quad . \tag{10}$$

It is evident that F_{12} thus defined obeys the constraint in Equation 6.

The additional equation to derive the coefficient F_{66} can be established according to Pouyet *et al.* [16] by completing an off-axis test applying tensile stress equal to the strength U_ϕ in the bias direction ϕ. By expressing all the resulting principal tensile and shear stresses σ_L, σ_T, and τ_{LT} in terms of U_ϕ and ϕ, Equation 7 can be expanded into

$$U_\phi[F_{11} \cos^4 \phi F_{22} \sin^4 \phi + (2F_{12} + F_{66}) \sin^2 \phi \cos^2 \phi] = 1 \quad , \tag{11}$$

from which the coefficient F_{66} can be determined. Then, fabric shear strength S can be readily evaluated as [11]

$$S = \frac{1}{\sqrt{F_{66}}} \quad . \tag{12}$$

Since S cannot be negative, this relation, combined with Equations 4 and 5, implies that the coefficients F_{11}, F_{22}, and F_{66} must possess positive values. Calculated results for the five fabrics are provided in Table IV, including predictions of fabric shear strength S.

TABLE IV. Coefficients of the strength tensors of the fabrics.

	Fabric 1	Fabric 2	Fabric 3	Fabric 4	Fabric 5
F_{11}, 10^{-4}	5.18	26.43	14.03	23.59	22.31
F_{22}, 10^{-4}	7.59	24.65	13.10	37.87	24.83
F_{12}, 10^{-4}	−3.13	−12.76	−6.78	−14.94	−11.77
F_{66}, 10^{-4}	68.59	168.93	83.52	143.69	120.60
X, 9.8 · N	43.95	19.45	26.70	20.59	21.17
Y, 9.8 · N	36.30	20.14	27.63	16.25	20.07
S, 9.8 · N	12.08	7.69	10.94	8.34	9.11

Recall that the values of X and Y can be predicted theoretically using our earlier results [14]. If we can somehow predict off-axial tensile strength U_ϕ as well, we can then derive fabric shear strength S based on the yarn properties and fabric structure without relying on uniaxial fabric tensile tests.

In Equations 11 and 12, note that the fabric shear strength S predicted from the equations will not necessarily be the same when U_ϕ tested at a different direction ϕ is substituted, due to the strength anisotropic nature of woven fibers. This is shown clearly in Table V, where the shear strengths S_ϕ for fabric 1 as a function of both U_ϕ and ϕ are calculated, and they show a nonmonotonic relation with both U_ϕ and ϕ. This again is likely caused by fabric structural irregularities and yarn-yarn relative movement during tensile tests in bias directions, discussed earlier. Unfortunately, we don't have any justification at present to accept or reject any one of the five predictions.

TABLE V. Effect of ϕ on the prediction of fabric shear strength (for fabric 1).

ϕ	15°	30°	45°	60°	75°
U_ϕ, 9.8 · N	34.27	35.73	23.08	18.01	14.51
S_ϕ, 9.8 · N	11.97	18.27	12.08	8.27	3.89
S_ϕ/U_ϕ	0.35	0.51	0.52	0.46	0.27

There have been several experimental attempts [12, 20], using the tested off-axis tensile properties at $\phi = 45°$, to estimate fabric shear properties. Equations 11 and 12, however, indicate that although shear strength is related to off-axis tensile strength, the relation is not single-valued. Table V also contains the ratios S_ϕ/U_ϕ of the predicted shear strength and corresponding off-axis tensile strength. If we use this ratio as an indicator of the closeness between S_ϕ and U_ϕ, we can conclude that $\phi = 45°$, which yields the highest S_ϕ/U_ϕ ratio, is probably the optimal direction at which to test off-axis tensile strength to approximate the shear behavior of the fabric. Also, we can conclude from the data that the shear strength of woven fabrics is lower than its lowest tensile strength at any direction.

Conclusions

The tensile strength anisotropy of a woven fabric can be approximated by a harmonic expression, and the accuracy of the approximation can be improved by increasing the number of harmonic components in the expression. The first constant harmonic component corresponds to the isotropic level of fabric tensile strength. The first two lower order components depict an elliptical shape of the failure surface, which resembles those of ordinary solid anisotropic materials. Yet for woven fabrics, higher order components are necessary to depict inherit irregularities in the fabrics. Anisotropic characteristics also exist in fabric breaking strains and tensile moduli. The loci of these mechanical properties, however, are generally not similar.

By assuming that the Tsai-Wu failure criterion is valid, at least in the first quadrant in a stress plane where both the stresses in the fabric principal directions are tensile, fabric shear strength can be obtained using the theory. Fabric shear strength is lower than fabric tensile strength tested at any direction, but off-axial tensile strength at $\phi = 45°$ is the closest approximation of predicted shear strength compared to tensile strengths at any other direction.

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