Development of a Constitutive Theory for Short-fiber Yarns
Part IV: The Mechanics of Blended Fibrous Structures*

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Received 13.3.1995 Accepted for publication 4.5.1995

This paper presents a thorough study on the prediction of the elastic-material constants ratios of blended short-fiber yarns, based on the theory developed by the author for yarns of a single fiber type by considering the structures as transversely isotropic and combined with the techniques used in dealing with hybrid composite materials. The mean tensile strengths of the blended staple-fiber yarns are also predicted, with the exclusion of the statistical and hybrid effects, which will be dealt with in a separate paper because of their extreme complexity. The so-called minimum blend ratio, below which the mean yarn strength will not follow the rule of mixtures, and the critical blend ratio, below which the mean yarn strength will be weakened rather than strengthened owing to the addition of the reinforcing fibers, are calculated, and the variables involved are discussed. Finally, the effects of the breaking strains between the blending-fiber types on the yarn properties are investigated. All results are schematically illustrated and the necessary parametric studies are provided.

1. INTRODUCTION

Fiber blending has for a long time been practiced in textile processing. By mixing fibers of different types to form textile yarns, many advantages are achieved, such as property compensation or reinforcement between fibers, cost reduction without significant sacrifice of yarn performance by partially replacing expensive fibers with less expensive ones, and cross-dyeing effects due to the different dye affinity of two fiber types. Owing to the importance of blended-yarn structures, it is desirable to understand and specify their mechanical behavior in order to realize the potential of the blending process.

Investigation of the mechanics of blended-fiber yarns has been the topic of many studies [1–6], and most of these studies are focused on the prediction of the tensile strength of the yarn as the most important yarn property with practical significance. In view of the complexity of the mechanics of staple-fiber yarns even with a single fiber type, the presence of two fiber types in a structure adds a formidable dimension to the theoretical analysis.

There are several aspects that make hybrid structures much more difficult to analyze. In hybrid yarns in which two different types of fiber are blended together to form a system, the difference in their contributions towards the over-all behavior of the structure, due to the diverse mechanical properties of the constituent fibers, has to be considered. Secondly, the interaction between the two constituents will alter the nature of yarn behavior, especially during fracture. Inclusion of this interaction in analysis has been proven to be very challenging. One phenomenon associated with this interaction in a blended structure that

*Parts I–III were published in *Textile Research Journal.*

greatly complicates analysis is the so-called 'hybrid effect', defined as a positive or negative deviation of a certain mechanical property from the rule-of-mixtures behavior [7].

The present author and his colleagues [8] have carried out a study on the computer modeling of the strength and fracture behavior of blended continuous-filament yarns. Based on such concepts as chain-of-subbundle, the changing lateral constraint between filaments due to twist and its effect on filament strength, and the load-sharing process between the broken and still-surviving members during yarn breakage, a new statistical approach was proposed, and a discrete computer model was introduced to predict the strength and fracture behavior of a blended continuous-filament yarn. Recently, we have also completed a study [9] trying to explain the cause of the 'hybrid effect' in a blended yarn.

On the other hand, research on the so-called hybrid composites, materials made by combining two or more different types of fiber in a common matrix, has become increasingly active [7,10–13]. The similarities between the two structures are many, except that, in a textile yarn, fibers are bound together through a frictional mechanism brought about by twist, whereas, in a composite, it is the chemical bonding that causes fibers to adhere to the matrix material. Many approaches and techniques in the hybrid-composite area can hence be applied with necessary modifications to the study of blended yarns.

To begin with, this paper deals only with the prediction of the elastic properties of blended yarns. The present study is intended to look into the problems in the mechanics of blended staple-fiber yarns on the basis of the results of the property predictions provided by the author previously [14–17] for staple-fiber yarns made of a single fiber type. Using these predictions and the approaches employed in hybrid-composite studies, we can calculate the corresponding blended-yarn properties for a given blend ratio and yarn twist and given fiber types and fiber orientations. Some of the yarn properties, such as the shear moduli in the longitudinal and transverse directions and the tensile modulus in the transverse direction, although very important parameters in studying yarn behavior and the behavior of fabrics made of the yarns, are very difficult, if not impossible, to determine experimentally, and the theoretical prediction seems to be the only convenient means to obtain them.

In addition, the mean yarn tensile strength is predicted in this study according to the rule of mixtures. The more difficult issues, such as the interactions between the fibers, the local stress redistribution due to fiber breakage, the hybrid effect, and the statistical aspects, are excluded. These important yet complex problems have been analyzed by Pan and Postle in a separate paper [9]. The minimum and the critical blend ratios of the reinforcing fiber are determined, and the effects of fiber-breaking strains are also investigated. In addition, investigations are carried out in this study to examine the effects of the blend ratio, yarn-twist level, and fiber properties on these system parameters.

2. THE ELASTIC-MATERIAL CONSTANTS OF A BLENDED STAPLE-FIBER YARN

First of all, as theoretically demonstrated by the present author [15], the effect of the fiber slippage at fiber ends in staple-fiber yarns during yarn extension becomes negligible when the yarn-twist level is reasonably high. Hence the fiber-slippage influence will be excluded in the present study, where focus on the practical twist range is adequately high. Interested readers can refer to the earlier paper [15] for treatment of the fiber-slippage effect at a low yarn-twist level.

The key variable in determining the properties of a hybrid fibrous system is the concentration of each fiber type in the system. There are two slightly different indices to specify the fiber concentration. The first one is the fiber-volume fraction, and the second one, more commonly used in the textile industry, is the blend ratio. If the yarn we are dealing with is a blended yarn with blend ratio B_a for fiber type a and B_b for type b, the corresponding fiber-volume fraction V_a for fiber type a is then given by

$$V_a = B_a V_f \tag{1}$$

and V_b for fiber type b by:

$$V_b = B_b V_f \tag{2}$$

Since $B_a + B_b = 1$, so that

$$V_a + V_b = V_f \tag{3}$$

where V_f is the total fiber-volume fraction of the yarn, which is a function of the yarn-twist factor T_y, and for a particular yarn studied previously [4], we have:

$$V_f = 0.7\left(1 - 0.78e^{-0.195T_y}\right) \tag{4}$$

It has become widely accepted that the elastic moduli for hybrid structures follow the rule-of-mixtures behavior [18], that is, the resultant properties of the structure are the mean values of the volume-fraction-weighted properties of the constituents. More specifically, for properties in the longitudinal directions, the rule of mixtures can be used directly, whereas, for those in the transverse directions, the inverse rule of mixtures has to be applied. The elastic properties of a blended yarn can thus be predicted by using the results provided previously [14] of the elastic properties for a yarn of a single fiber type. Of the several elastic properties predicted [14] for a yarn treated as a transverse isotropy, the four major constants are shown here. If fiber type a as the reinforcing fiber is blended with fiber type b, the longitudinal tensile modulus of a blended yarn can be expressed in terms of the blend ratio for fiber a as:

$$E_L = \left[B_a E_a + (1 - B_a)E_b\right]V_f \eta_l \eta_{1\theta} \tag{5}$$

where E_a and E_b are the tensile elastic moduli for fiber types a and b, respectively, and

$$\eta_l = 1 - \frac{\tanh(ns)}{ns} \tag{6}$$

is called the length-efficiency factor, reflecting the effect of a definite staple-fiber length, where s is the so-called fiber-aspect ratio, $s = l_f/2r_f$, the ratio of the fiber length to its diameter. The parameter n is called the cohesion factor, an indicator of the gripping effect of the yarn structure on each individual fiber, and was defined [14] as:

$$n = \sqrt{\frac{G_{TL}}{E_f}\frac{2}{\ln 2}} \tag{7}$$

where G_{TL} is the yarn longitudinal shear modulus and E_f the fiber tensile modulus. For the hybrid case, the over-all value of the fiber tensile modulus can be calculated by using the values of each component as:

$$E_f = \left[B_a E_a + (1 - B_a)E_b\right]V_f \tag{8}$$

Moreover,

$$\eta_{1\theta} = \frac{3}{4}\frac{(1+\cos 2q)^2}{4\left(1+\cos 2q+\cos^2 2q\right)} \tag{9}$$

is the so-called fiber-orientation-efficiency factor representing the effect due to fiber alignment or obliquity in the yarn. The parameter q is the nominal yarn-surface helix angle defined by Hearle [19] as:

$$q = \arctan\left[10^{-3} T_y \sqrt{\left(\frac{40\pi}{\rho_f V_f}\right)}\right] \tag{10}$$

where ρ_f is the fiber specific weight.

For convenience of discussion, Equation (5) can be rewritten in the form of a ratio of the moduli as:

$$\frac{E_L}{E_b} = \left[B_a \frac{E_a}{E_b} + (1-B_a)\right] V_f \eta_l \eta_{1\theta} \tag{11}$$

This ratio is an indicator of the reinforcing effect of fiber a on the tensile stiffness of the blended yarn. The transverse tensile modulus follows the so-called inverse rule of mixtures as:

$$\frac{1}{E_T} = \frac{B_a V_f}{E_{aT}} + \frac{(1-B_a)V_f}{E_{bT}} \tag{12}$$

where from earlier work [14]

$$E_{iT} = V_f E_i \eta_l \eta_{2\theta}, \qquad (i = a, b) \tag{13}$$

is the transverse tensile modulus of the yarn made of a single fiber type i, and $\eta_{2\theta}$ is the fiber-orientation-efficiency factor associated with the yarn transverse direction, given by:

$$\eta_{2\theta} = \frac{8}{\pi^2}\frac{(1/2q - 1/4\sin 2q)^2}{\left(2/3 - \cos q + 1/3\cos^3 q\right)(1-\cos q)} \tag{14}$$

Again, this result can be expressed in the form of a modulus ratio:

$$\frac{E_b}{E_T} = \frac{E_b}{E_a}\frac{B_a}{\eta_l \eta_{2\theta}} + \frac{(1-B_a)}{\eta_l \eta_{2\theta}} \tag{15}$$

Similarly, the longitudinal shear modulus is:

$$\frac{1}{G_{TL}} = \frac{B_a V_f}{G_{aTL}} + \frac{(1-B_a)V_f}{G_{bTL}} \tag{16}$$

where from earlier work [14]

$$G_{iTL} = V_f E_i \eta_l \eta_{12\theta}, \qquad (i = a, b) \tag{17}$$

and the corresponding orientation-efficiency factor

$$\frac{1}{\eta_{12\theta}} = S(T,L) = \frac{\pi(1-\cos q)\sin^3 q}{6(1/2q - 1/4\sin 2q)^2} + \frac{8\sin^3 q}{3\pi(1-\cos q)(1+\cos q)^2} + \frac{\pi\left(4 - 3\cos q - \cos^3 q\right)}{6(1/2q - 1/4\sin 2q)(1+\cos q)} \tag{18}$$

Alternatively, Equation (16) can be expressed in the modulus-ratio form:

$$\frac{E_b}{G_{TL}} = \frac{E_b}{E_a}\frac{B_a}{\eta_l \eta_{12\theta}} + \frac{(1-B_a)}{\eta_l \eta_{12\theta}} \tag{19}$$

The major (in the L–T direction) Poisson's ratio of the blended yarn is given by:

$$\upsilon_{LT} = B_a V_f \upsilon_{aLT} + (1-B_a) V_f \upsilon_{bLT} \tag{20}$$

where we also have from earlier work [14] that:

$$\upsilon_{iLT} = \frac{\sin^{5q}}{2\left(1-\cos^3 q\right)(1/2q - 1/4\sin 2q)}, \qquad (i = a, b) \tag{21}$$

As indicated in Equation (21), the yarn Poisson's ratio is determined by the yarn-surface helix angle q alone, independently of the fiber mechanical properties when we ignore the Poisson's effects of the fibers themselves. We therefore have:

$$\upsilon_{aLT} = \upsilon_{bLT} \tag{22}$$

or

$$\upsilon_{LT} = (V_a + V_b)\upsilon_{aLT} = \upsilon_{aLT} \tag{23}$$

that is, fiber blending will not alter the Poisson's ratios of the yarn if we neglect the fiber Poisson's effects.

It has to be pointed out here that, as previously demonstrated by the present author [16], the yarn-surface helical angle defined in Equation (10) was derived on the basis of a filament structure. Modifications are therefore necessary when applying it to a staple-fiber-yarn structure. It was also shown [16] that a satisfactory prediction of the well-known fiber-obliquity effect in the yarn longitudinal tensile modulus can be achieved if $2q$ instead of q is used in Equation (9), as has already been done; this observation, however, was found to be valid only for the yarn longitudinal tensile properties so that no such modification is needed in equations relating other properties.

Furthermore, to simplify the present results for practical applications, both fiber-orientation factors, associated with the longitudinal tensile modulus defined in Equation (9) and with the Poisson's ratio in Equation (21), can be approximated by a function $\cos^2 q$.

The comparisons before and after the simplification, provided in Figures 1 and 2, respectively, show reasonably close agreements between the pairs. With regard to the orientation factors associated with the longitudinal shear modulus and transverse tensile modulus, although it was shown previously [14] that, when the yarn-twist factor increases, both moduli follow paths similar to the exponential functions and approach their own asymptote as a result of three such competing factors as the fiber-volume fraction, the fiber-orientation-efficiency factor, and the fiber-length-efficiency factor, it is difficult to find simple approximations for the orientation-efficiency factors.

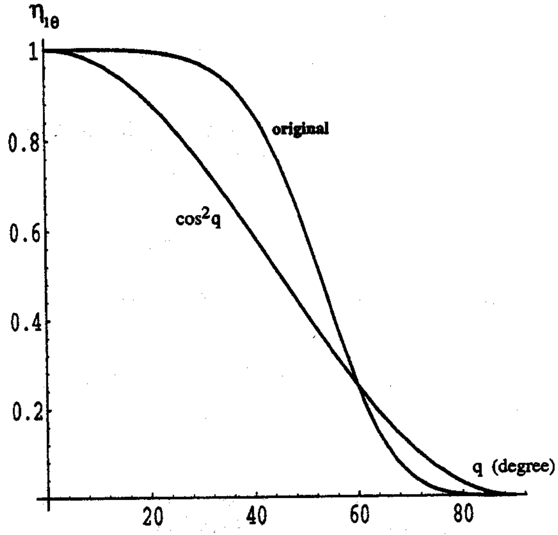

Fig. 1 Comparison of the original and simplified orientation-efficiency factors *I*

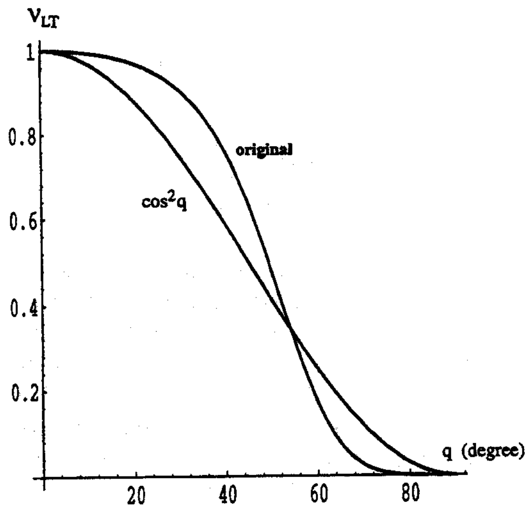

Fig. 2 Comparison of the original- and simplified-orientation yarn Poisson's ratios V_{LT}

3. TENSILE STRENGTHS OF BLENDED STAPLE-FIBER YARNS: A NON-PROBABILISTIC APPROACH

As mentioned before, the strength prediction of a blended staple-fiber yarn is a task of tremendous complexity, since it is dependent on many statistical factors of a high degree of uncertainty, such as the structural variations and the interactions between the distinct fiber constituents in the yarn. For instance, since the fiber distribution is not constant along the yarn cross-sections, blended staple-fiber yarns from even the same package will possess different strengths. Yet, for most industrial applications, the statistical mean strength can still provide helpful indications of the yarn quality and durability.

To simplify the problem further, we start with a yarn in which each fiber is assumed to be of uniform strength along its length. We also assume linear stress–strain relations for both fiber a and fiber b such that their tensile moduli are constants. Suppose further that fiber type a has a lower breaking elongation than type b. As a result, the ultimate tensile strength σ_{yu} of a blended yarn can again be predicted on the basis of the rule of mixtures as:

$$\sigma_{yu} = \left(\sigma_{au} V_a + \varepsilon_{au} E_b V_b\right)\eta_l \eta_{l\theta} \tag{24}$$

where σ_{au} is the ultimate strength and ε_{au} is the failure strain of the fiber type a.

It has already been reported by Coplan [1], on the basis of his empirical speculation, that three corrections have to be made in order to account for the differences between the fiber strength and the experimentally determined strength of a yarn made of that fiber. The first correction is due to fibers falling in the open fringe of the yarn cross-section. The second correction is caused by the effect of the 'weakest-link', and the third accounts for the fiber obliquity due to the fiber-helix orientation. The theory developed by the present author [14] has, in fact, theoretically derived the factors corresponding to each correction. The fiber-volume fraction $V_f < 1$ represents the first correction, the fiber-length-efficiency factor $\eta_l < 1$ reflects the 'weakest-link' effect, and the fiber-obliquity effect is accounted for by the fiber-orientation-efficiency factor $\eta_l < 1$. More importantly, this theory has also established the interconnection between these three factors and pointed out that the yarn-twist factor T_y is the key variable that affects the contributions of all three factors.

Equation (24) can be further expressed in terms of the blend ratio B_a as:

$$\sigma_{yu} = \left[\sigma_{au} B_a + \varepsilon_{au} E_b \left(1 - B_a\right)\right] V_f \eta_l \eta_{l\theta} \tag{25}$$

or in the form of a strength ratio:

$$\frac{\sigma_{yu}}{\sigma_{au}} = \left[B_a + \frac{\varepsilon_{au} E_b}{\sigma_{au}}\left(1 - B_a\right)\right] V_f \eta_l \eta_{l\theta} = \left[B_a + \frac{E_b}{E_a}\left(1 - B_a\right)\right] V_f \eta_l \eta_{l\theta} \tag{26}$$

Here, owing to the linear fiber stress–strain relationship, $\sigma_{au}/\varepsilon_{au} \equiv E_a$ has been applied.

4. THE MINIMUM AND CRITICAL BLEND RATIOS FOR THE REINFORCING FIBER

By blending a new fiber type (the reinforcing component) with the existing fiber, we could improve the properties of the final product. However, this property improvement will not be achieved unless the amount of the reinforcing component exceeds a certain limit. There are actually two such limits, which are the characteristic values of a specific system. Let us take the yarn strength as an example. The conclusions, of course, are valid for other properties.

Similarly to the treatment in a hybrid composite, we can derive a minimum fiber-blend ratio B_{amin} for fiber type a. If the amount of the reinforcing component, B_a, is too small to be taken into account, the yarn strength will become:

$$\sigma_{yu} = \sigma_{bu}(1 - B_a)V_f \eta_l \eta_{1\theta} \tag{27}$$

where σ_{bu} is the ultimate strength of fiber type b.

Equating Equations (25) and (27) yields the minimum fiber-blend ratio B_{amin} for the reinforcing component:

$$B_{amin} = \frac{\sigma_{bu} - \varepsilon_{au}E_b}{\sigma_{au} + \sigma_{bu} - \varepsilon_{au}E_b} = \frac{\dfrac{\sigma_{bu}}{\sigma_{au}} - \dfrac{E_b}{E_a}}{1 + \dfrac{\sigma_{bu}}{\sigma_{au}} - \dfrac{E_b}{E_a}} \tag{28}$$

Here $\sigma_{au}/\varepsilon_{au} \equiv E_a$ has again been used. It is indicated in this equation that the minimum fiber-blend ratio for the reinforcing component is independent of the fiber-volume fraction, fiber size, and yarn twist (fiber-orientation-efficiency factor). Also note that the rule of mixtures, Equation (25), is valid only when $B_a \geq B_{amin}$.

Furthermore, Equations (25) and (27) show that the strength of a blended yarn, even with the existence of reinforcing fiber a, can be lower or higher than the strength of the yarn made of the weaker fiber type b (see Equation (32)) alone. The strength of the blended yarn will be higher only when the blend ratio of fiber a exceeds a critical value B_{acrit}, whose value can be determined by the following.

In order to make

$$\sigma_{yu} = [\sigma_{au}B_a + \varepsilon_{au}E_b(1 - B_a)]V_f\eta_l\eta_{1\theta} \geq \sigma_{bu}V_f\eta_l\eta_{1\theta} \tag{29}$$

the critical blend ratio of fiber a is thus deduced as:

$$B_{acrit} = \frac{\sigma_{bu} - \varepsilon_{au}E_b}{\sigma_{au} - \varepsilon_{au}E_b} = \frac{\dfrac{\sigma_{bu}}{\sigma_{au}} - \dfrac{E_b}{E_a}}{1 - \dfrac{E_b}{E_a}} \tag{30}$$

It is easy to see that always

$$1 > B_{acrit} \geq B_{amin} \geq 0 \tag{31}$$

This gives us from Equations (30) and (31) the restraints on the fiber properties, such as:

$$\frac{\sigma_{bu}}{\sigma_{au}} \geq \frac{E_b}{E_a}, \qquad E_b < E_a, \qquad \sigma_{bu} < \sigma_{au} \tag{32}$$

These conditions have to be met in order to realize the reinforcement. More specifically, in order to reinforce a structure, the reinforcing component has to possess not only a higher breaking strength but also a higher tensile modulus than the existing component. It may also be seen from Equations (28) and (30) that the ratios of the strength, σ_{bu}/σ_{au}, and tensile modulus, E_b/E_a, of the two fibers are the key parameters in determining the reinforcing effects.

5. CALCULATION AND DISCUSSION

The fiber properties used for calculation are listed in Table I. For convenience, we assume that both fibers have the same radius, length, and specific density ρ_f as listed.

Table I
The Fiber Properties Used for Calculation

Item	Typical Value	Unit
Fiber radius r_f	3×10^{-3}	cm
Fiber length l_f	3.0	cm
Fiber aspect ratio $s = \frac{l_f}{2r_f}$	500	
Fiber specific density ρ_f	1.31	g/cm^3

By using Equations (4), (8), (9), (10), (18), and (23), the related yarn parameters at three levels of the twist factor T_y are calculated and shown in Table II.

Table II
The Yarn Parameters Predicted for Other Calculations

$T_y\sqrt{tex}$ turns/cm	20	30	40
q (deg)	13.28	19.37	25.10
η_l	0.980	0.984	0.990
$\eta_{1\theta}$	0.947	0.890	0.820
$\eta_{12\theta}$	0.098	0.139	0.168
V_f	0.689	0.698	0.700
ν_{LT}	0.988	0.971	0.944

First, let us examine the relationship between the modulus ratio E_L/E_b and the blend ratio B_a at three levels of the twist factor T_y. By using Equation (11), this relationship can be plotted as shown in Fig. 3, where fiber a is taken as the reinforcing fiber with a higher tensile modulus, $E_a = 2E_b$. As has been shown earlier, the value of the modulus ratio E_L/E_b is an indicator of the reinforcing effect on the yarn tensile stiffness. It can be concluded from Fig. 3 that the yarn longitudinal tensile modulus ranges from below $0.6E_b$ to over $1.2E_b$, depending mostly on the blend ratio (the amount of the reinforcing fiber a) as well as on the yarn-twist level. At a given twist level, increasing the blend ratio B_a will result as expected in a stiffer yarn, whereas, at a given blend ratio and yarn-twist range, higher twist will cause a higher fiber-obliquity effect and lead to a lower yarn modulus. In any case, however, the yarn modulus cannot reach $2E_b$ or E_a: that is, the yarn naturally cannot be stiffer than the reinforcing fiber.

Similar discussion can be applied to the yarn shear modulus G_{TL}. Fig. 4 is plotted according to Equation (19) to show the relationship between the modulus ratio G_{TL}/E_b and the blend ratio B_a at three levels of the twist factor T_y. It may be seen from the figure that the yarn longitudinal shear modulus is much smaller than its longitudinal tensile modulus as shown in Fig. 3. There is a non-linear relation between G_{TL} and B_a, compared with a linear one between E_L and B_a. A similar effect is seen of the blend ratio B_a on the yarn shear modulus. The function of yarn twist is that a higher twist level always tightens the structure and hence increases the yarn shear modulus.

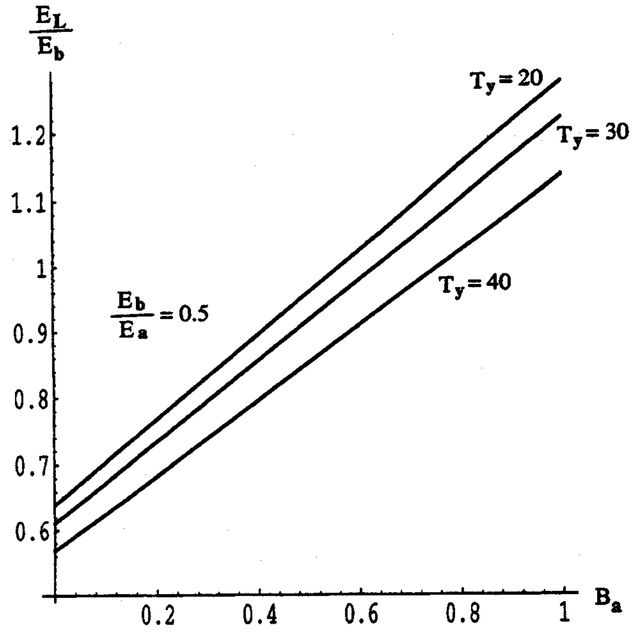

Fig. 3 Yarn tensile modulus v. blend ratio at three twist levels

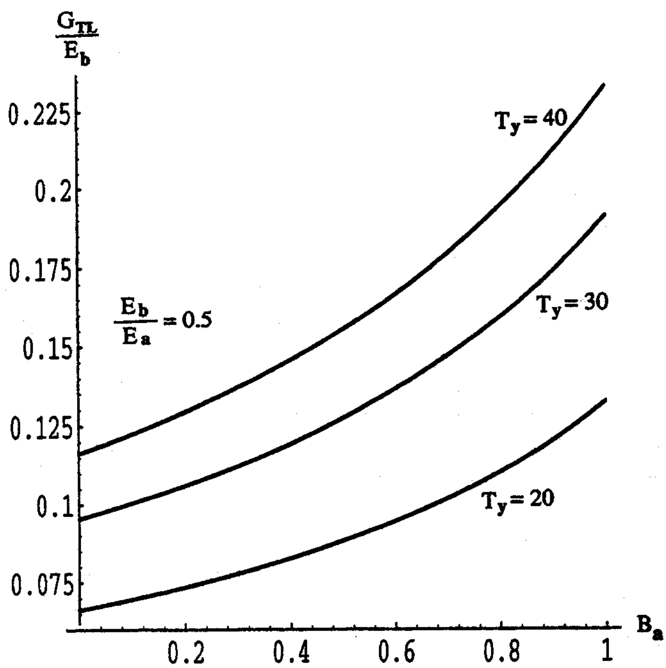

Fig. 4 Yarn shear modulus v. blend ratio at three twist levels

Fig. 5, illustrating Equation (26), shows the importance of both the fiber modulus and the blend ratio in reinforcing the yarn tensile strength. To evaluate the effect of the blend ratio B_a, the figure can be divided into two different zones at the location $E_b/E_a = 1$. Let us consider the fiber type with the higher modulus as the reinforcing component here. In the region where $E_b/E_a < 1$ or $E_a > E_b$, type a is the reinforcing fiber. As a result, a higher blend ratio B_a leads to a stronger yarn. However, in the region where $E_b/E_a > 1$ or $E_a < E_b$, type b becomes the reinforcement and a higher B_a, meaning less of fiber b, will weaken the yarn. This again indicates the fundamental reinforcing mechanism that a higher tensile modulus is crucial in fiber-reinforcing practice. On the other hand, once the blend ratio B_a is given and fiber b is now the reinforcement, increasing the modulus ratio E_b/E_a will result in a stronger yarn.

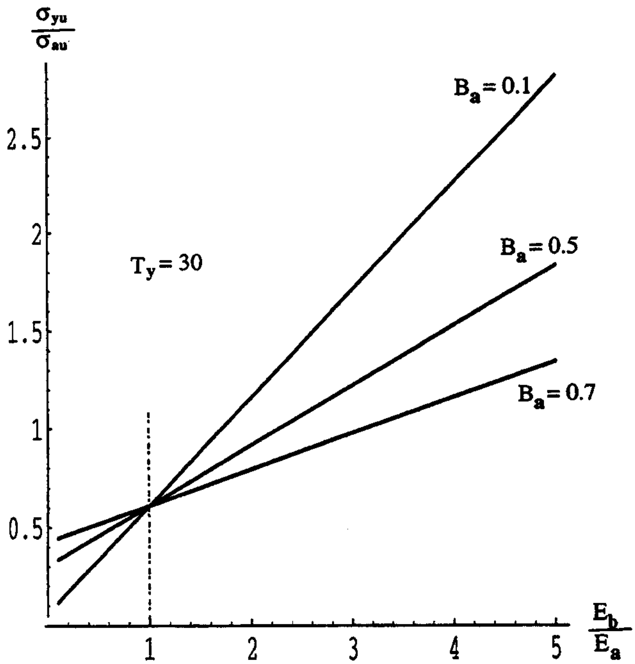

Fig. 5 Yarn breaking strength v. fiber-modulus ratio at three blend levels

Fig. 6 is provided to help understanding of the physical meanings of both the minimum and critical blend ratios of the reinforcing component. Curves 1–3–5 and 2–3–4 are also plotted by using Equation (26), where Curve 2–3–4 is the second part; hence Equation (26) is represented by the curve 2–3–5 collectively. The rule of mixtures that governs the blended-yarn behavior follows the 3–5 line section. It is therefore seen that the blend ratio of fiber a has to exceed B_{amin} before the rule of mixtures becomes applicable. In addition, point 2 represents the strength of a yarn made of 100% fiber b. The line segment 2–3 shows that, when very few fibers of type a as the reinforcement are blended into the structure, the yarn is weakened rather than strengthened. It is not until the amount of fiber a reaches the critical value B_{acrit} that the yarn starts to become stronger than the yarn of 100% fiber b.

Figures 7 and 8 show the influence of related fiber properties on the values of B_{amin} and B_{acrit}, respectively. Fig. 7 is constructed on the basis of Equation (28) to look into the effects on B_{amin} of the fiber-modulus ratio E_b/E_a and the fiber-strength ratio σ_{bu}/σ_{au}. It is shown in this figure that, in the present range, the fiber-strength ratio σ_{bu}/σ_{au} is a more important parameter in affecting the value of B_{amin}, for B_{amin} will remain zero as long as the fiber-strength ratio is lower than a certain value, say, 0.2 in the figure, regardless of the range of the fiber-modulus ratio E_b/E_a. In other words, when the reinforcement is much stronger than the existing component, the minimum blend ratio will disappear so that the rule of mixtures will be applicable over the full range of the blend ratio B_a. Yet, for a reasonably high fiber-strength ratio, the value of B_{amin} varies depending on the fiber-modulus ratio; a higher modulus ratio will lead to a smaller B_{amin} value. That is, the more different the moduli of the two fiber types are, the more reinforcement is needed to make the rule of mixtures applicable.

Fig. 8, on the other hand, is based on Equation (30) to show the relations between both E_b/E_a and the strength ratio σ_{bu}/σ_{au} and the critical blend ratio B_{acrit}. It depicts similar trends to those observed in Fig. 7 except that this time all the relations are linear. Again, a higher σ_{bu}/σ_{au} value leads to a higher B_{acrit}, meaning that more reinforcement is needed before the system is strengthened. For a yarn with a given strength ratio, a higher E_b/E_a value will result in a small B_{acrit}; that is, choosing two fiber types with similar tensile moduli will require a smaller amount of the reinforcing fiber to make a stronger yarn.

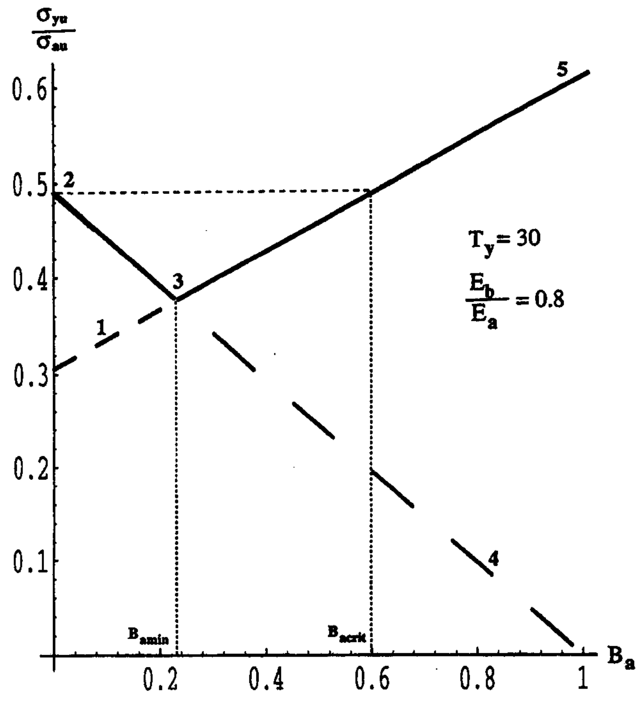

Fig. 6 Yarn breaking strength v. blend ratio and the minimum and critical blend ratios

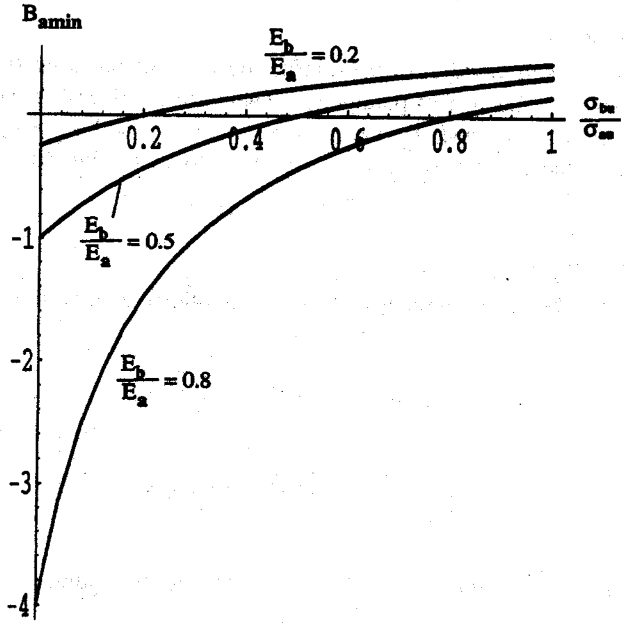

Fig. 7 Minimum blend ratio v. fiber-strength ratio at three levels of fiber-modulus ratio

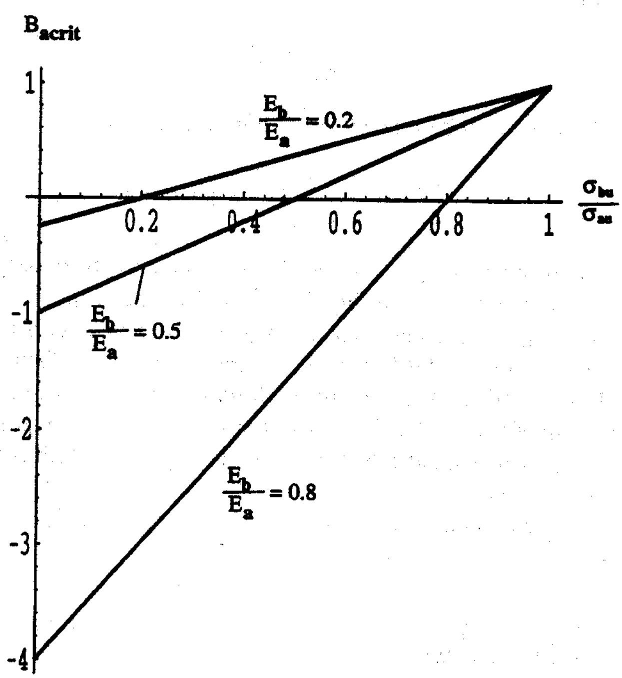

Fig. 8 Critical blend ratio v. fiber-modulus ratio at three twist levels

6. EFFECTS OF THE BREAKING STRAIN OF THE REINFORCING FIBER

6.1 Possible Relations

Besides the strength and modulus of the reinforcing fiber, its breaking strain ε_{au} also has a profound influence on the system parameters. There are in all three possibilities in terms of the relation between ε_{au} and ε_{bu}, i.e. $\varepsilon_{au} < \varepsilon_{bu}$, $\varepsilon_{au} = \varepsilon_{bu}$, and $\varepsilon_{au} > \varepsilon_{bu}$. The preceding analyses are all based on case 1: $\varepsilon_{au} < \varepsilon_{bu}$. We shall focus on the other two cases in this section.

6.2 Effects when $\varepsilon_{au} = \varepsilon_{bu}$

In this case, both fiber a and fiber b will break simultaneously, so that the yarn strength will become:

$$\sigma_{yu} = \left[\sigma_{au}B_a + \sigma_{bu}(1-B_a)\right]V_f\eta_l\eta_{1\theta} \tag{33}$$

or

$$\frac{\sigma_{yu}}{\sigma_{au}} = \left[B_a + \frac{\sigma_{bu}}{\sigma_{au}}(1-B_a)\right]V_f\eta_l\eta_{1\theta} \tag{34}$$

Since there is always $\frac{\sigma_{au}}{\varepsilon_{au}} \equiv E_a$ and $\frac{\sigma_{bu}}{\varepsilon_{bu}} \equiv E_b$, we can hence obtain:

$$\frac{\varepsilon_{bu}}{\varepsilon_{au}} = \frac{\sigma_{bu}}{\sigma_{au}}\frac{E_a}{E_b} \tag{35}$$

and bringing in $\varepsilon_{au} = \varepsilon_{bu}$ yields:

$$\frac{\sigma_{bu}}{\sigma_{au}} = \frac{E_b}{E_a} \tag{36}$$

Thus Equation (34) can also be related to the fiber-modulus ratio:

$$\frac{\sigma_{yu}}{\sigma_{au}} = \left[B_a + \frac{E_b}{E_a}(1-B_a)\right]V_f\eta_l\eta_{1\theta} \tag{37}$$

The relation between yarn strength and the fiber-modulus (or fiber-strength) ratio as well as the blend ratio B_a can be seen in Fig. 9, which illustrates Equation (37) (or Equation (34)). The terms $V_f\eta_l\eta_{1\theta}$ are constant here and will not influence our discussion, so that we set $V_f\eta_l\eta_{1\theta} = 1$ when plotting the following figures. In the case when $\varepsilon_{au} = \varepsilon_{bu}$, the yarn strength will always increase when more reinforcing fiber is blended into the structure. The level of either the fiber-modulus or fiber-strength ratio is also important: it determines the initial yarn strength as well as the rate at which the yarn strength increases when more reinforcing fiber is added.

Following the same derivations, we can prove in this case that the minimum blend ratio for fiber type a is:

$$B_{amin} = 0 \tag{38}$$

and that the critical blend ratio is:

$$B_{acrit} = 0 \tag{39}$$

That is, Equation (33), the rule of mixtures, will now be applicable for the entire range of $0 \le B_a < 1$, and the blended yarn will always be stronger than the yarn made of fiber type b alone, no matter how small an amount of reinforcing fiber a is blended in. This is also illustrated clearly in Fig. 9.

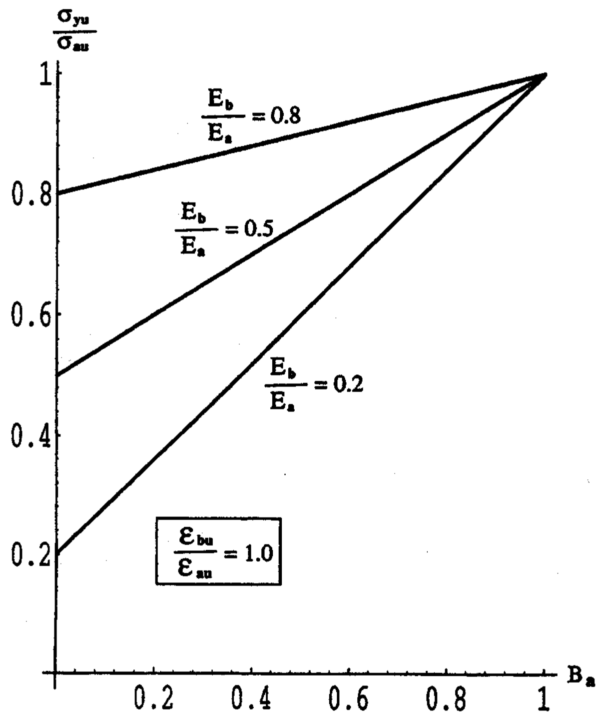

Fig. 9 Yarn-strength ratio v. fiber-blend ratio at three fiber-modulus-ratio levels

6.3 Effects when $\varepsilon_{au} > \varepsilon_{bu}$

In this case, we have:

$$\sigma_{yu} = \left[\varepsilon_{bu} E_a B_a + \sigma_{bu}(1 - B_a)\right] V_f \eta_l \eta_{l\theta} \tag{40}$$

or

$$\frac{\sigma_{yu}}{\sigma_{au}} = \left[\frac{\varepsilon_{bu} E_a}{\sigma_{au}} B_a + \frac{\sigma_{bu}}{\sigma_{au}}(1 - B_a)\right] V_f \eta_l \eta_{l\theta} = \left[\frac{\varepsilon_{bu}}{\varepsilon_{au}} B_a + \frac{\sigma_{bu}}{\sigma_{au}}(1 - B_a)\right] V_f \eta_l \eta_{l\theta} \tag{41}$$

Because of the condition $\varepsilon_{au} > \varepsilon_{bu}$, we now have a restriction on the selection of the fiber properties based on Equation (35), i.e.:

$$\frac{\varepsilon_{bu}}{\varepsilon_{au}} = \frac{\sigma_{bu}}{\sigma_{au}} \frac{E_a}{E_b} < 1 \tag{42}$$

Fig. 10 is thus plotted at three fiber-strain-ratio levels by using Equation (41), where we choose the fiber-strength ratio $\sigma_{bu}/\sigma_{au} = 0.4$. Then, to meet the restriction in Equation (42), the fiber-modulus ratio has to be $E_a/E_b > 0.4$. Fig. 10 does show that the ratio of the fiber-breaking strain has a considerable effect on the fiber-reinforcing function. It is seen that, when the fiber-breaking-strain ratio $\varepsilon_{bu}/\varepsilon_{au}$ is greater than 0.4 in this case, increasing the proportion of the reinforcing fiber, type a, will strengthen the yarn. Yet increasing B_a will weaken the yarn when $\varepsilon_{bu}/\varepsilon_{au}$ is lower than 0.4. The yarn strength becomes independent of the value of B_a when $\varepsilon_{bu}/\varepsilon_{au}$ = 0.4.

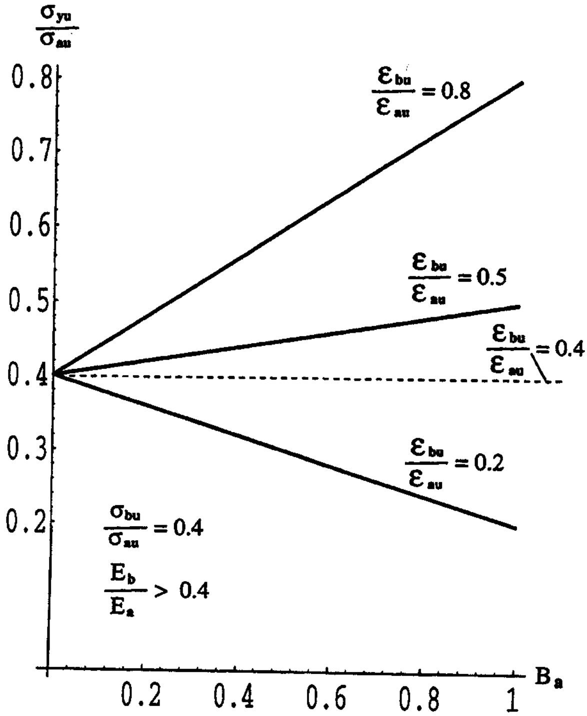

Fig. 10 Yarn-strength ratio v. fiber-blend ratio at three fiber-strain-ratio levels

Over-all, the fiber-strength ratio σ_{bu}/σ_{au} and fiber-modulus ratio E_a/E_b are the two key variables in the fiber-reinforcing effect. First, the two ratios have to satisfy the restriction in Equation (42) collectively so that, once a ratio is selected to be at a certain level, then the other ratio cannot take an arbitrary value. Secondly, the level (say, 0.4) of either ratio is critical, since it determines the level of the fiber-breaking-strain ratio $\varepsilon_{bu}/\varepsilon_{au}$ at which the yarn strength will keep a constant initial value and is no longer a function of the blend ratio B_a, as may be seen in Fig. 10.

It can also be easily proved that, in this case, the issues of the minimum and critical blend ratios are again no longer existent.

7. CONCLUSIONS

The elastic-material constants of a blended yarn can be derived by using the analysis provided in this study. The key variables in determining these constants include the blend ratio, the fiber size, the fiber orientation, the fiber modulus, and the fiber-volume fraction of each component and, most of all, the yarn-twist level, which determines many of the factors. The Poisson's ratio of the blended yarn, however, is proven to be identical to that of a single-fiber yarn, independently of the blend ratio if the fiber Poisson's effect is ignored.

The mean strength of a blended yarn is strongly influenced by the ratios of the fiber modulus, fiber strength, and fiber-breaking strain. This influence is better characterized by using the minimum and critical blend ratios of the reinforcement. The minimum blend ratio and critical blend ratio exist only when the breaking strain of the reinforcement is lower than that of the existing fiber type. Otherwise, whether the blended yarn strength will increase along with the amount of the reinforcing fiber will be dependent on the fiber-breaking strains. If both fiber types have an identical breaking strain, the yarn strength will always be stronger than the strength of the yarn made of fiber type b alone, no matter how small an amount of reinforcing fiber a is incorporated. In the case in which the reinforcing fiber is more extensible, the yarn strength can become higher or lower or remain constant when the amount of the reinforcing fiber increases, depending on the difference between the fiber-breaking strains of the two fiber types.

ACKNOWLEDGMENT

The author wishes to express his thanks for the reviewer's comments, which have led to a significant improvement of the last section of the paper.

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